TM-6011 - Modul 5: Formulasi Numerik (Model Diskrit)

Advanced Reservoir Simulation

Finite Difference Equations - A Discrete Approach

Discrete Approach

- Does not derive equation from the partial differential equation through Taylor series (continues approach)
- Continuous approach is not commonly used in practice
- Now, we will learn how to derive the equation through block (discretization of partial differential equations)

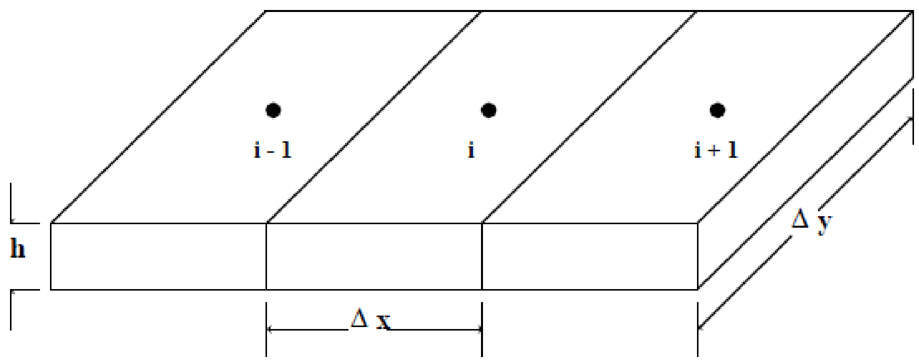
Oil Material Balance Equation

- Begin with a statement of the continuity, or material balance, equation:

Net rate of Flow in (scf/D) = Rate of Accumulation (scf/D)

Oil Material Balance

- Consider a 1-D of homogeneous model:



- The pore volume of the each gridblock is:
  $V_p = \Delta x \, \Delta y \, h \, \phi$ and

$$OIP = \frac{V_p S_o}{B_o}$$

- The Rate of accumulation during the time step, Δt is:

$$\frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_0}\right)^{n}\right]$$

$q_{right} \rightarrow$	i	$\leftarrow q_{left}$

- Net rate of flow in = $q_{left} + q_{right}$

- **Net rate of Flow in (scf/D) = Rate of Accumulation (scf/D)**

$$q_{lft} + q_{rht} = \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]$$

- Flow from the right, from gridblock i+1 to gridblock i, is:

$$q_{rht}(scf / D) = \frac{u_{rht} A}{B_o}$$

or

$$q_{right} = \left(\frac{0.00633kk_{ro}}{\mu_o B_o}\right)\left(\frac{p_{i+1} - p_i}{\Delta x}\right)(\Delta y h)$$

$$= \left(\frac{0.00633kh\Delta y}{\Delta x}\right)\left(\frac{k_r}{B\mu}\right)_o (p_{i+1} - p_i)$$

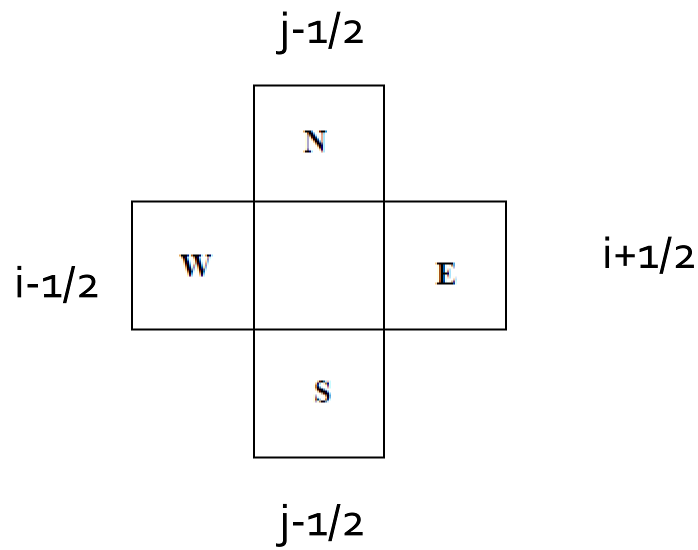
Introducing New Term

- Transmissibility, T = k A/L

$$T_{i+1/2} = T_E = \frac{0.00633 k h \Delta y}{\Delta x}$$

- Mobility, $k_r/(\mu B)$

$$M_{i+1/2} = \left(\frac{k_r}{B\mu}\right)_{o\, i+1/2} = \left(\frac{k_r}{B\mu}\right)_{oE}$$



$$(B_o)_E = (B_{o\,i} + B_{o\,i+1})/2$$

$$(\mu_o)_E = (\mu_{oi} + \mu_{oi+1})/2$$

$$(k_{ro})_E = \text{upstream } k_{ro}$$

Terms Consolidation

- Introducing a_{oE} (mobility x transmissibility)

$$a_{oE} = \left(\frac{k_r}{B\mu}\right)_{oE} T_E$$

Oil Material Balance
Final discretization

- For 1-D problem:

$$
\begin{aligned}
& a_{oE}(p_{i+1} - p_i) \\
+ \; & a_{oW}(p_{i-1} - p_i) \;=\; \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]
\end{aligned}
$$

- For 2-D problem:

$$
\begin{aligned}
& a_{oE}(p_{i+1} - p_{ij}) \\
+ \; & a_{oW}(p_{i-1} - p_{ij}) \\
+ \; & a_{oS}(p_{j+1} - p_{ij}) \\
+ \; & a_{oN}(p_{j-1} - p_{ij}) \;=\; \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]
\end{aligned}
$$

Oil Material Balance
Final discretization

- For 3-D problem:

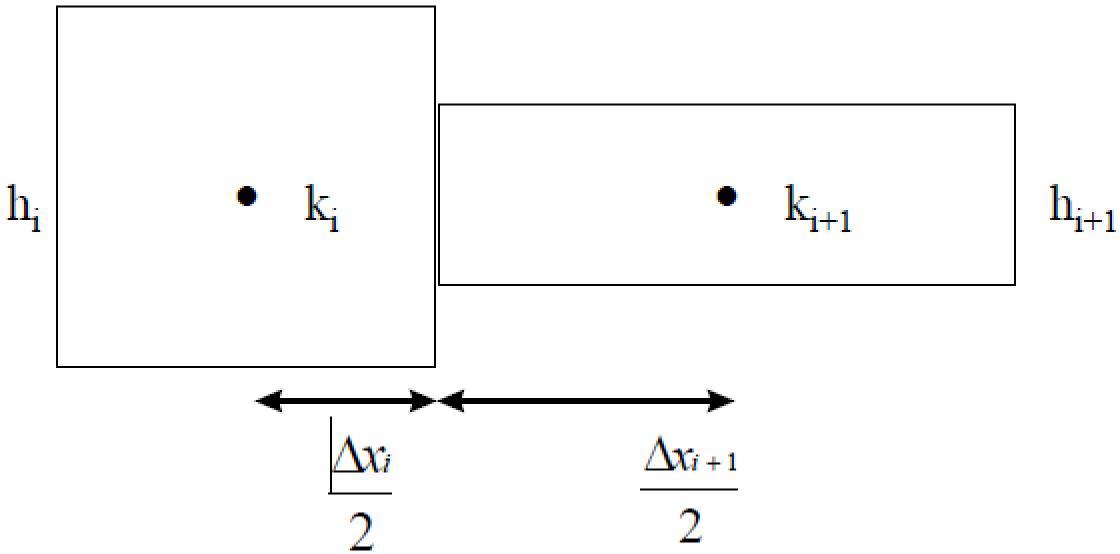
$$
\begin{aligned}
& a_{oE}(p_{i+1} - p_{ijk}) \\
+ \quad & a_{oW}(p_{i-1} - p_{ijk}) \\
+ \quad & a_{oS}(p_{j+1} - p_{ijk}) \\
+ \quad & a_{oN}(p_{j-1} - p_{ijk}) \\
+ \quad & a_{oB}(p_{k+1} - p_{ijk}) \\
+ \quad & a_{oT}(p_{k-1} - p_{ijk}) = \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]
\end{aligned}
$$

Simplification with "difference" operator

- The Oil Material Balance is:

$$\Delta a_o \Delta p = \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]$$

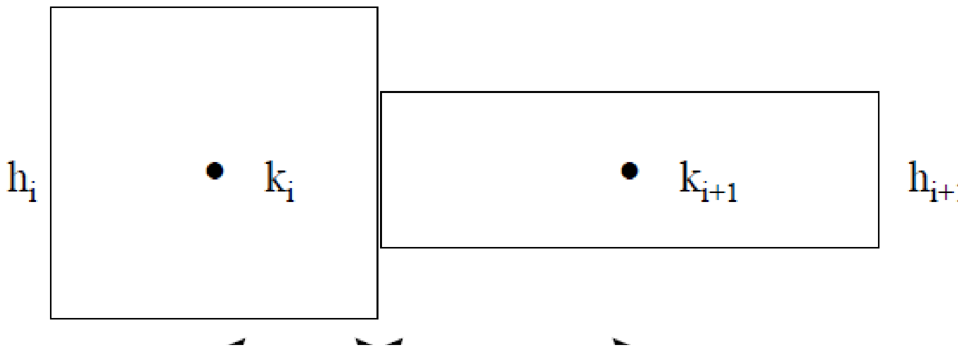
Heteregenous Reservoir



$$T_1 = 0.00633 \frac{k_i h_i \Delta y}{\Delta x_i / 2} \qquad T_2 = 0.00633 \frac{k_{i+1} h_{i+1} \Delta y}{\Delta x_{i+1} / 2}$$

$$q = \left(\frac{k_r}{B\mu}\right)_{oE} T_1 (p_{i+1/2} - p_i) = \left(\frac{k_r}{B\mu}\right)_{oE} T_2 (p_{i+1} - p_{i+1/2}) = \left(\frac{k_r}{B\mu}\right)_{oE} T_E (p_{i+1} - p_i)$$

Heteregenous Reservoir



Since

$$p_{i+1} - p_i = (p_{i+1/2} - p_i) + (p_{i+1} - P_{i+1/2})$$

$$q \Big/ \left(\frac{k_r}{B\mu}\right)_{oE} T_1 + q \Big/ \left(\frac{k_r}{B\mu}\right)_{oE} T_2$$

$$q\left(\frac{1}{T_1} + \frac{1}{T_2}\right) \Big/ \left(\frac{k_r}{B\mu}\right)_{oE} = q\left(\frac{T_1 + T_2}{T_1 T_2}\right) \Big/ \left(\frac{k_r}{B\mu}\right)_{oE}$$