TM-6011 - Modul 4: Formulasi Numerik (Model Kontinu)

Advanced Reservoir Simulation

Reservoir Flow Forces

Naturally, there are 3 forces work in the reservoir:

$$Viscous\ Force = \frac{\mu}{0.00633k} u,\ psi/ft$$

$$Gravity\ Force = \frac{\Delta\rho}{144},\ psi/ft$$

$$Capillary\ Force = \nabla P_c = \frac{dP_c}{dS} \nabla S,\ psi/ft$$

Investigation of Reservoir Forces

1. If there is no flow then viscous force = 0.

2. If there is no density difference (single phase, for example) then gravity force = 0.

3. If there is no saturation gradient (single phase, for example) then capillary force = 0.

Gravity Number= the ratio of gravity and viscous forces

Finite Difference Equations - A Continuous Approach

WHY?

- not possible to develop analytical solutions for many problems
- Heterogeneity and irregular shapes of reservoirs.
- The nonlinearity of the equations (properties changing with pressure)

Taylor Series

$$p(x+\Delta x) = p(x) + \Delta x p'(x) + \frac{\Delta x^2}{2!}p''(x) + \frac{\Delta x^3}{3!}p'''(x) + \bullet\bullet\bullet + \frac{\Delta x^n}{n!}p^n(x) + \bullet\bullet\bullet$$

- where p^n is the n th derivative of p. This is an infinite series which is theoretically exact for an infinite number of terms.
- if we truncate the series after n terms, then we introduce a *truncation error* (the remaining terms which are not included)

$$p_{i+1} = p_i + \Delta x\frac{\partial p}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 p}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 p}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 p}{\partial x^4} + \bullet\bullet\bullet$$

$$p_{i-1} = p_i - \Delta x\frac{\partial p}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 p}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 p}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 p}{\partial x^4} - \bullet\bullet\bullet$$

Finite Difference Schemes Approximation of $\delta p/\delta x$

Forward difference

$$\frac{\partial p}{\partial x} \approx \frac{p_{i+1} - p_i}{\Delta x} + \vartheta(\Delta x)$$

Backward difference

$$\frac{\partial p}{\partial x} \approx \frac{p_i - p_{i-1}}{\Delta x} + \vartheta(\Delta x)$$

Central difference

$$\frac{\partial^2 p}{\partial x^2} \approx \frac{p_{i-1} - 2p_i + p_{i+1}}{(\Delta x)^2} + \vartheta(\Delta x^2)$$

Finite Difference Scheme - Summary

<u>Difference Expression</u>

	p_{i-3}	p_{i-2}	p_{i-1}	p_i	p_{i+1}	p_{i+2}	p_{i+3}	
Forward Difference								
$\Delta x\, p'(x)$				-1	1			
$(\Delta x)^2\, p''(x)$				1	-2	1		
Backward Difference								
$\Delta x\, p'(x)$			-1	1				$+\, O(\Delta x)$
$(\Delta x)2\, p''(x)$		1	-2	1				
Central Difference								
$2\, \Delta x\, p'(x)$			-1	0	1			$+\, O(\Delta x)^2$
$(\Delta x)^2\, p''(x)$			1	-2	1			

High Order Finite Difference

	p_{i-3}	p_{i-2}	p_{i-1}	p_i	p_{i+1}	p_{i+2}	p_{i+3}	
Forward Difference								
$2\Delta x\, p'(x)$				-3	4	-1		$+ O(\Delta x)^2$
$(\Delta x)^2 p''(x)$				2	-5	4	-1	
Backward Difference								
$2\Delta x\, p'(x)$		1	-4	3				$+ O(\Delta x)^2$
$(\Delta x)^2 p''(x)$	-1	4	-5	2				
Central Difference								
$12\Delta x\, p'(x)$		1	-8	0	8	-1		$+ O(\Delta x)^4$
$12\,(\Delta x)^2 p''(x)$		-1	16	-30	16	-1		