IMPES Method

Finite Difference Equations

$$\boxed{a_{oE} = \left(\frac{Kr}{B\mu}\right)_{oE}\left(\frac{0.00633kh\Delta y}{\Delta x}\right)_E}$$

- Oil

$$\Delta\left(a_o^n \Delta p^{n+1}\right) = \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^{n}\right]$$

- Water

$$\Delta\left(\mathrm{a}_\mathrm{w}^\mathrm{n} \Delta \mathrm{p}^{\mathrm{n}+1}\right) = \frac{1}{\Delta \mathrm{t}}\left[\left(\frac{\mathrm{V_p S_w}}{\mathrm{B_w}}\right)^{\mathrm{n}+1} - \left(\frac{\mathrm{V_p S_w}}{\mathrm{B_w}}\right)^{\mathrm{n}}\right]$$

- Gas

$$\Delta\left(a_g^n \Delta p^{n+1}\right) = \frac{1}{\Delta t}\left[\left(\frac{V_p S_g}{B_g}\right)^{n+1} - \left(\frac{V_p S_g}{B_g}\right)^{n}\right]$$

IMPES Steps…

1) Calculate coefficients of the pressure equation
2) Calculate solution of the pressure equation implicitly (matrix equation) for: p^{n+1}
3) Calculate solution of the saturation equations explicitly for: S_o^{n+1}, S_w^{n+1}, and S_g^{n+1}

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- **4 unknowns per block:**

  p^{n+1}, S_o^{n+1}, S_w^{n+1}, and S_g^{n+1}
- **To find the unknowns, we need one more equation per block:** $S_o^{n+1} + S_w^{n+1} + S_g^{n+1} = 1$
- **Assures fluid volumes fit the pore volume**

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- Oil
- Water
- Gas

$$S_o^{n+1} = \frac{B_o^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_o}{B_o}\right)^n + \Delta t \cdot \Delta\left(a_o^n \Delta p^{n+1}\right)\right]$$

$$S_w^{n+1} = \frac{B_w^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_w}{B_w}\right)^n + \Delta t \cdot \Delta\left(a_w^n \Delta p^{n+1}\right)\right]$$

$$S_g^{n+1} = \frac{B_g^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_g}{Bg}\right)^n + \Delta t \cdot \Delta\left(a_g^n \Delta p^{n+1}\right)\right]$$

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Summing up all saturation equations:

$$\sum_{m=o,w,g} B_m^{n+1} \Delta\left(a_m^n \Delta p^{n+1}\right) =$$

$$\frac{1}{\Delta t}\left[V_p^{n+1} - \sum_{m=o,w,g} B_m^{n+1}\left(\frac{V_p S_m}{B_m}\right)^n\right]$$

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Approximate Vp^{n+1} on the right hand side using the identity:

$$V_p^{n+1} = V_p^n + \frac{\Delta V_p}{\Delta p}\left(p_i^{n+1} - p_i^n\right)$$

and a chord slope:

$$\frac{\Delta V_p}{\Delta p} = \frac{V_p^{n+1} - V_p^n}{p_i^{n+1} - p_i^n}$$

IMPES Method

Then,

$$V_p^{n+1} = V_p^n\left[1 + \frac{1}{V_p^n}\frac{\Delta V_p}{\Delta p}\left(p_i^{n+1} - p_i^n\right)\right]$$

$$= V_p^n\left[1 + c_f\left(p_i^{n+1} - p_i^n\right)\right]$$

where,

$$c_f \equiv \frac{1}{\phi}\frac{\Delta\phi}{\Delta p} = \frac{1}{V_p^n}\frac{\Delta V_p}{\Delta p}$$

Likewise,

$$B_m^{n+1} = B_m^n\left[1 - c_m\left(p_i^{n+1} - p_i^n\right)\right] \quad (m = o, w, g)$$

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Final equation can now be written as:

$$\Delta\left(a_t^n \Delta p^{n+1}\right) = \frac{V_p^n c_t}{\Delta t}\left(p_i^{n+1} - p_i^n\right)$$

$$where:$$

$$\Delta a_t = \sum_{m=o,w,g} B_m^{n+1} \Delta a_m$$

$$c_t = c_f + \sum_{m=o,w,g} c_m S_m^n$$