TM-6011

Advanced Reservoir Simulation

Gridding Considerations

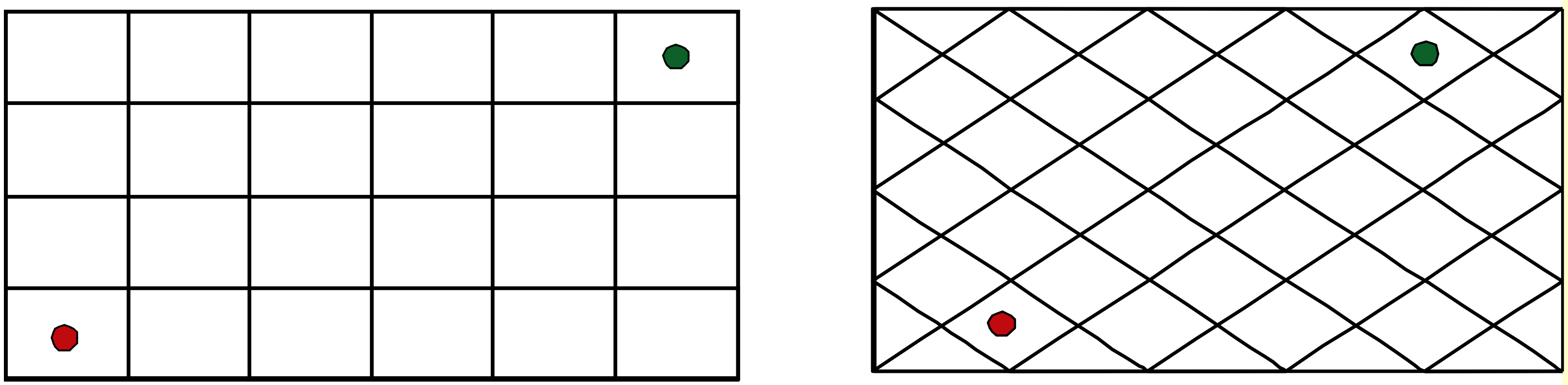
- Aziz (SPE 25233) lists the following considerations in choosing a gridding approach:
  - geology and reservoir size
  - type of displacement process
  - location and type of wells
  - desired numerical accuracy
  - available software
  - simulation study objectives
  - competence of simulation engineer/team
  - available computer resources

Gridding Options

- Modern simulators may offer the following options:
  - Local grid refinement
  - Hybrid grids
  - Curvilinear grid (streamline grid)
  - Voronoi/PEBI grid
  - Corner Point Grid
  - Dynamic Grid
  - Automatic Grid Generation

Grid Orientation

The orientation of the grid with respect to the well locations can influence the simulation results.



Fluid moves preferentially along the grid lines. If the red dot represented a water injector, the water would break through fastest in the diagonal grid (right) - even if the grid dimensions were the same.

Grid Orientation

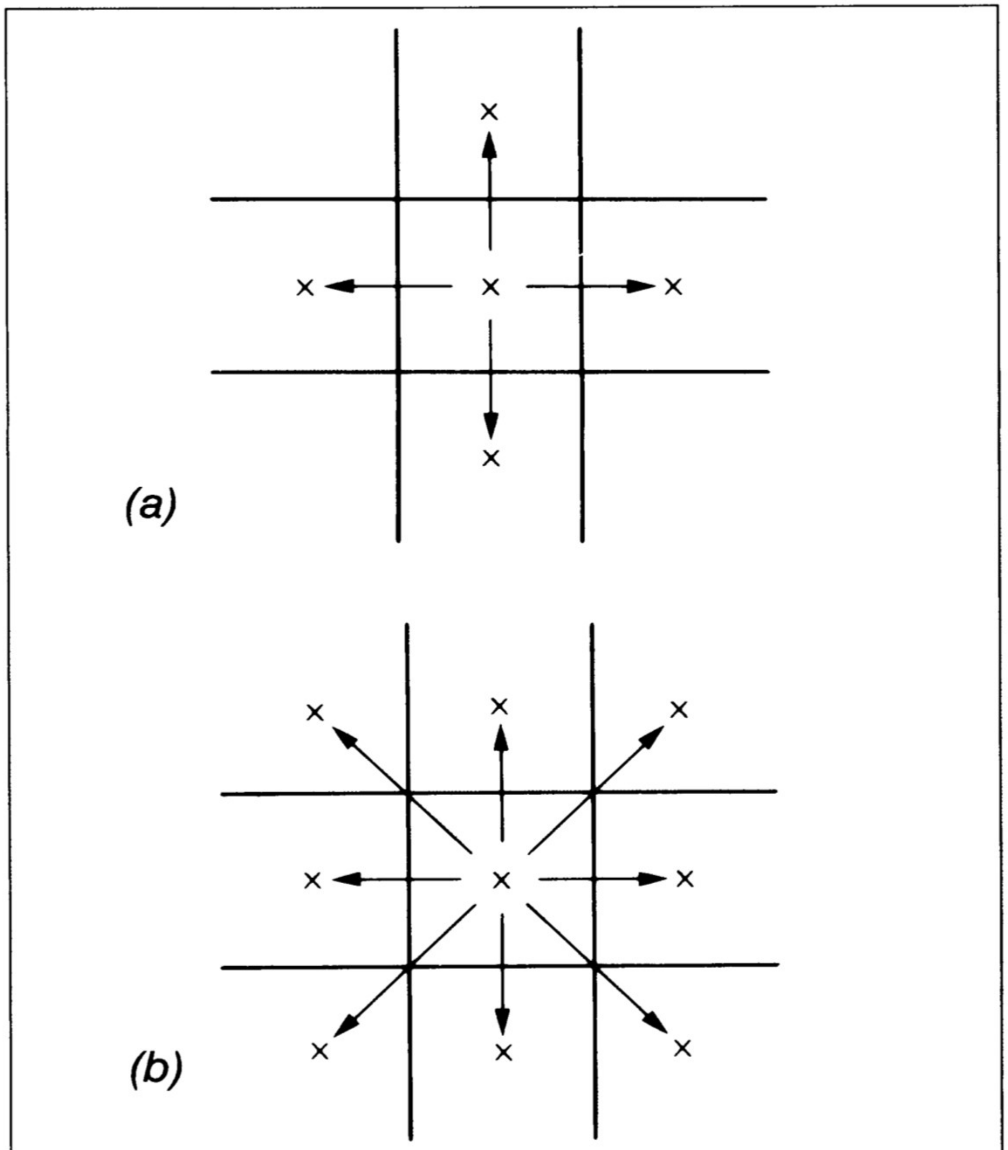


Fig. 5.20—Symbolic illustration of (a) five-point and (b) nine-point formulations of flow equations.

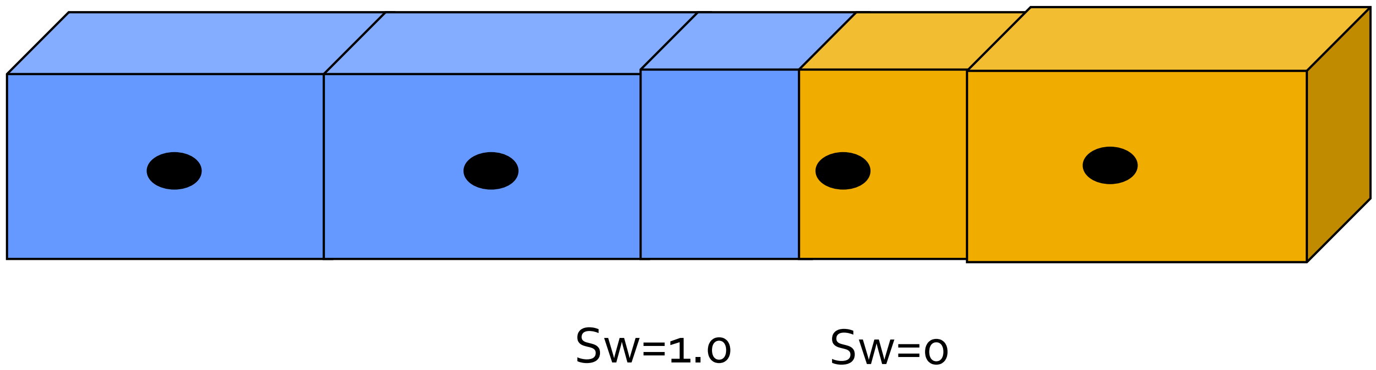
Mattax and Dalton

Numerical Dispersion

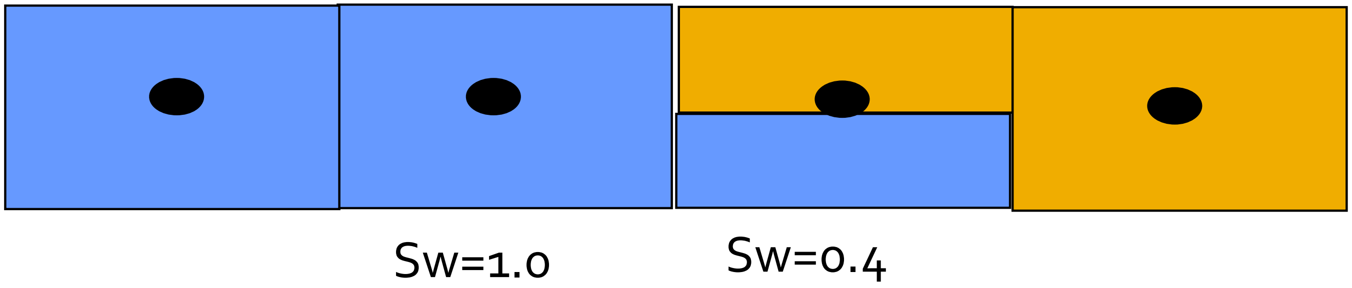
- Smearing of sharp fluid front.
- Physically in 1D waterflood water front advances forward by one cell *every* timestep.
- Problem occurs because simulator can not represent saturation distibution *within* a grid block.

Saturation Distribution, $t = t_1$

True Solution

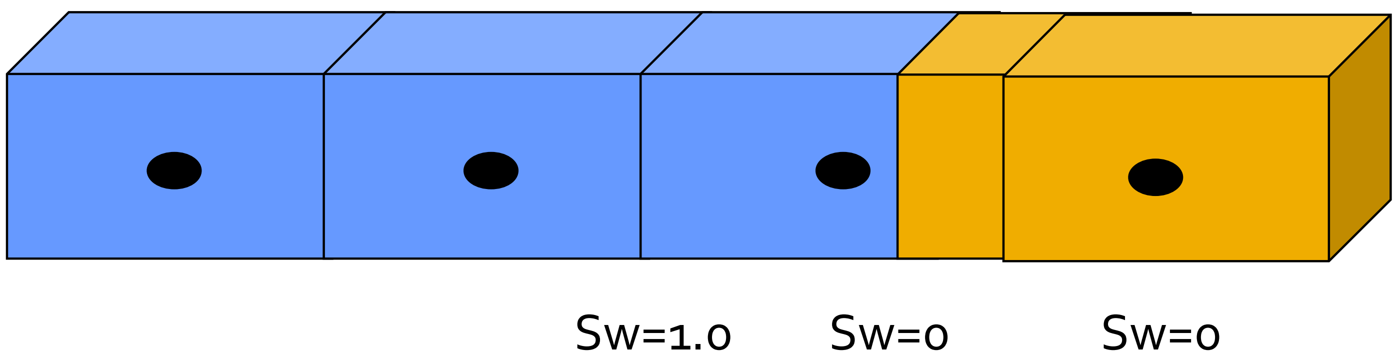


Simulator

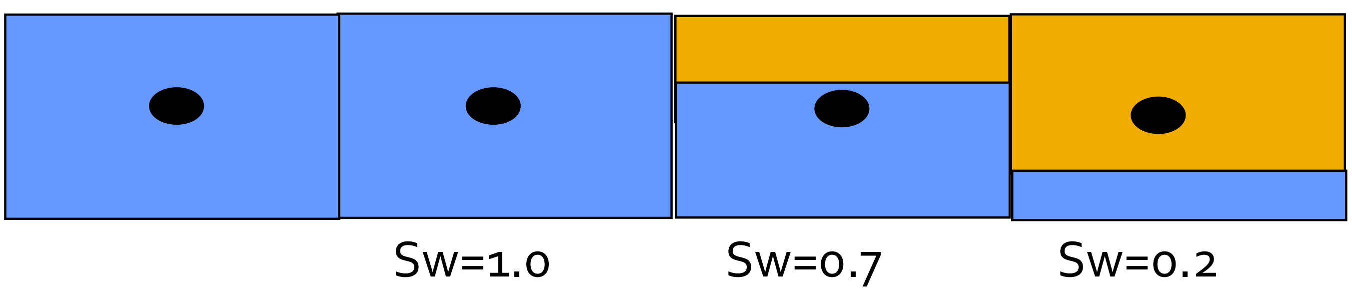


Saturation Distribution, $t = t_1 + \Delta t$

True Solution

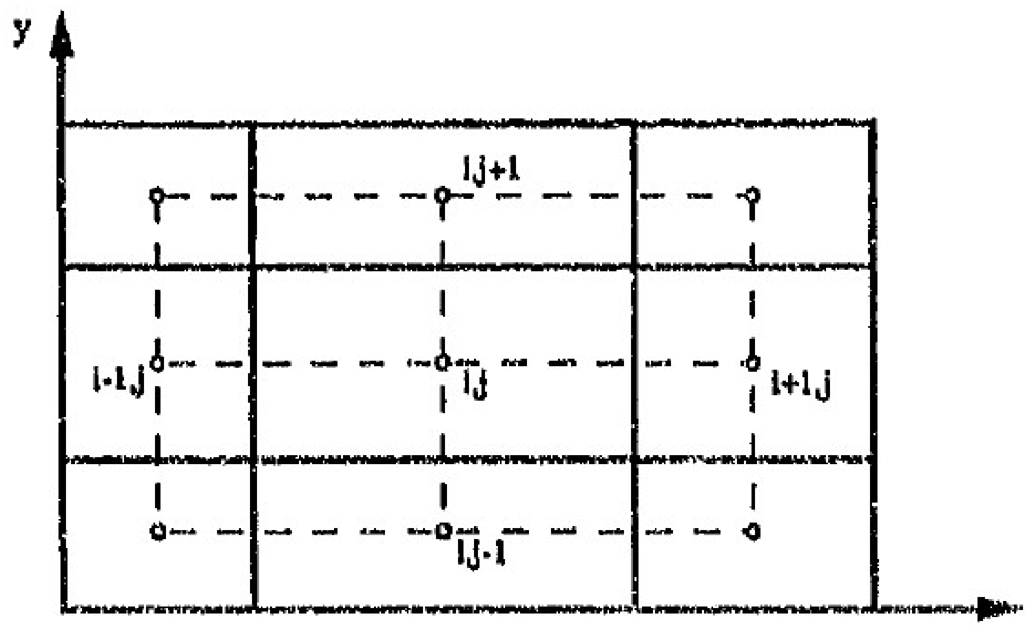


Simulator



Block-Centered Grids

- In a block-centered grid the reservoir is divided into blocks and nodes are located at the block centers.



a) Block-centered grid

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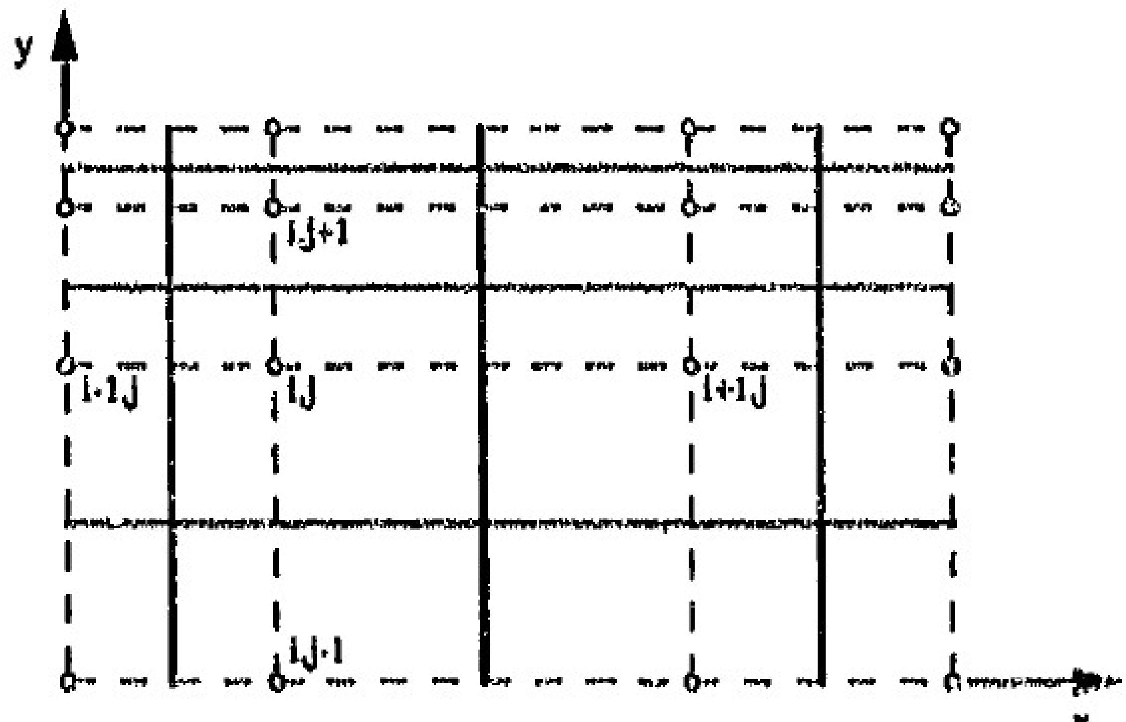
Block-Centered Grids

- Block-centered grids are the most commonly used.
- They are "familiar" approach for engineers.

Point-Distributed Grids

- In a point-distributed grid nodes are laid out first and then gridblocks are constructed around them.



b) Point-distributed grid

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Comparison

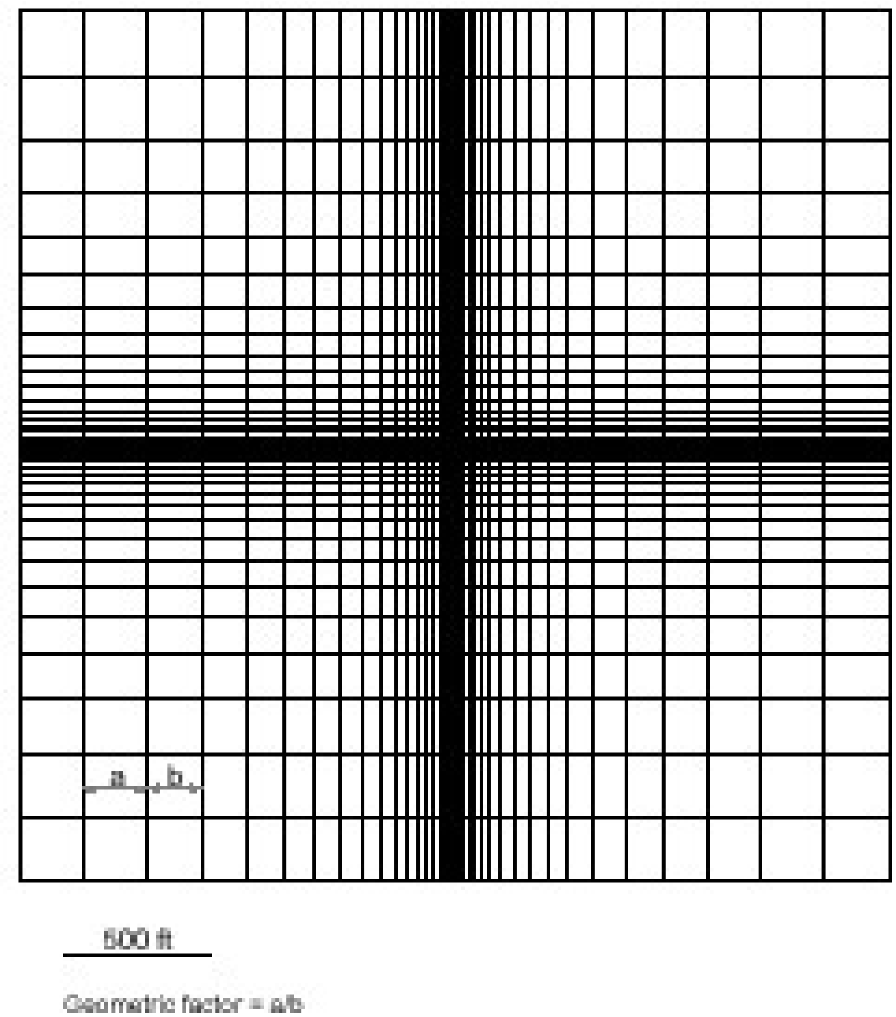
- The block-centred grid gives a more rigorous treatment of the accumulation term than the point-distributed grid.
- Conversely the point- distributed grid is more accurate for the flow terms.
- If the grid is regular (all cells same size) or close to it the difference between the methods is insignificant.

Local Grid Refinement

- “For large reservoir simulation problems, a fine grid is only needed in parts of the reservoir where saturations or pressure are changing rapidly.

- Using the standard irregular grid leads to unwanted small blocks in some parts of the reservoir.” (Aziz, SPE 25233).

Irregular Grid

An e: 

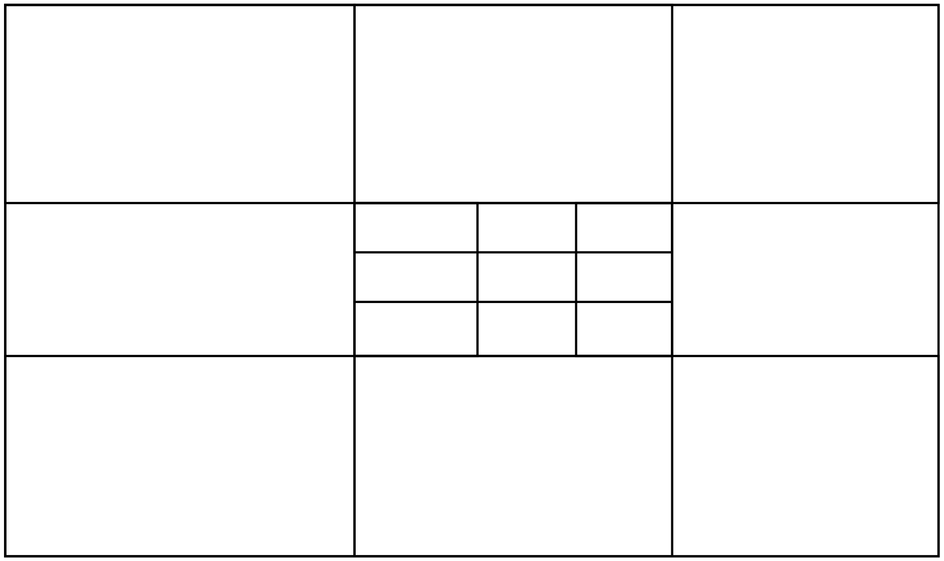
is:

The black region in the centre is a set of very fine grid cells.

(Archer and Horne, SPE 62916)

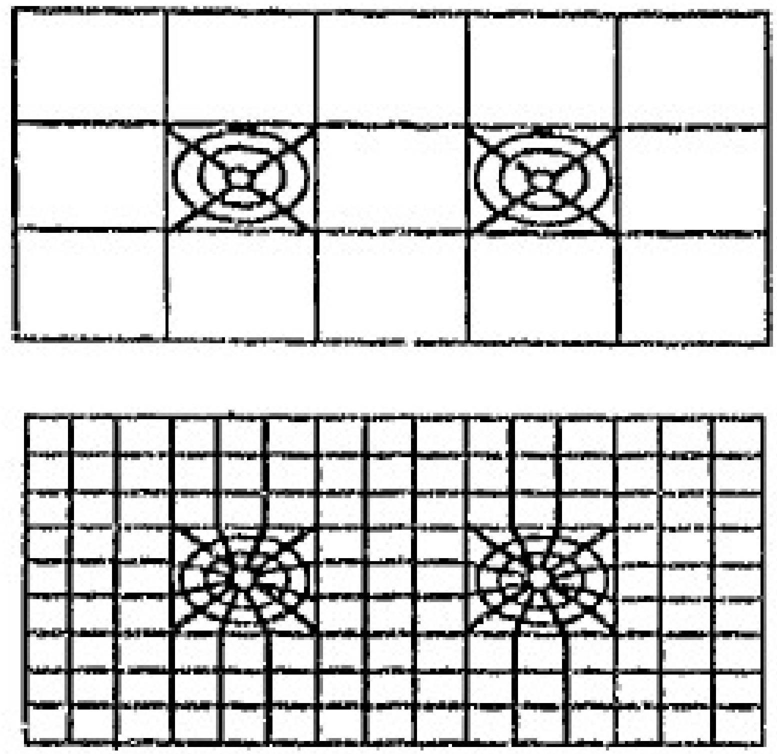
Local Grid Refinement

Local refinement divides a single cell (or set of cells) into multiple cells. The problem with this approach is accurate calculation of the flow between the coarse and fine blocks.



Hybrid Grids

- A hybrid grid is composed of two or more different types of grid which have been coupled together.



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Pros and Cons

1. Hybrid grids (and local grid refinements) help to resolve sharp saturation profiles near wells and improve the accuracy of water-oil ratio and gas-oil ratio calculations.

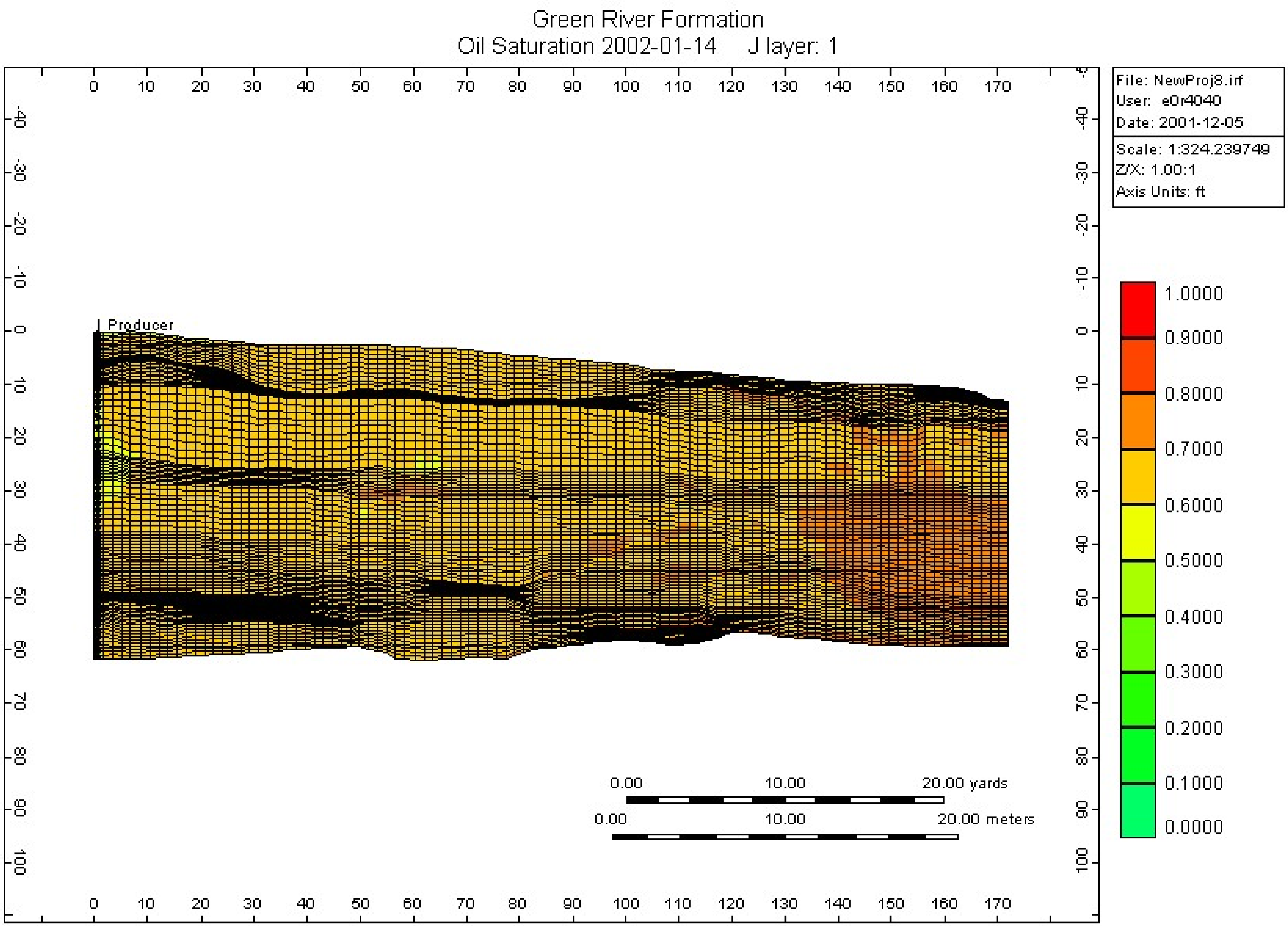
2. Banded structure of problem matrix is lost so matrix solving procedure may be more involved/less efficient.

Corner Point Gridding

- In a corner point grid the shape of the grid cells is completely irregular i.e. They are not regular “box” shapes. Usually the grid are “logically rectangular” i.e. they are still defined by 8 points (corner points).

- Corner point grid are used to resolve complex reservoir geometries.

Example



Enis Robbana, M.S.

Corner Point Gridding

- “The real problem with type of grid is that now flow across a block face depends on more than two pressures on either side of that face. … When the grid is skewed connection between blocks are no longer orthogonal to block faces.” (Aziz, SPE 25233)

Voronoi Gridding

- The concept of a Voronoi grid is a very old one. They were first defined in 1908.

- The used of these grids in reservoir simulation was pioneered by Dr. Heinemann at the University of Leoben.

Voronoi Gridding

- A Voronoi gridblock is defined as "the region of space that is closer to its gridpoint than to any other gridpoint". (Aziz, SPE 25233).
- The line between two gridpoints is <u>perpendicular</u> to the grid block boundary.
- The grid is a generalization of the point-distributed grid.

Streamline Grid

- In a “streamline grid” a grid for standard finite difference method is constructed by first solving for the streamline paths (by solving an IMPES pressure equation - we will return to streamline simulation)

- Then a corner point grid is constructed which follows the streamlines. Since the standard finite difference method is used to solve the flow equations fluid <u>can</u> move across streamlines.

Dynamic Grids

- In a dynamic grid the gridding is changed during the coarse of the simulation. This is a common technique in other branches of science and engineering.

- Grid cells are refined or coarsened depending on the speed of the flow at that point in time.

Can’tEscape from Equations….

Fully Implicit Method

- **Governing Equations**
- **Solution Technique**
- **Well Modeling**

Governing Equations

Material Balance Equation:

$$Accumulation = \frac{1}{\Delta t}\left[\left(\frac{V_p S}{B}\right)_e^{(n+1)} - \left(\frac{V_p S}{B}\right)_e^{n}\right]$$

V_p	= pore volume, *rcf*
S	= phase saturation, *fraction*
B	= formation volume factor, *rcf/scf*
e	= evaluated cell
D*t*	= time interval

Net Flux

$$\text{Net Flow Rate} = \sum_{i=1}^{N} q_{i\to e}, {}_{\mathrm{e}\neq \mathrm{i}}$$

For Explicit Scheme:

$$\sum_{i=1}^{N} q_{i\to e} = \sum_{i=1}^{N} T_{i\to e}\, M^{n}_{i\to e} \Delta\Phi^{n+1}_{i\to e}$$

For Implicit Scheme:

$$\sum_{i=1}^{N} q_{i\to e} = \sum_{i=1}^{N} T_{i\to e}\, M^{n+1}_{i\to e} \Delta\Phi^{n+1}_{i\to e}$$

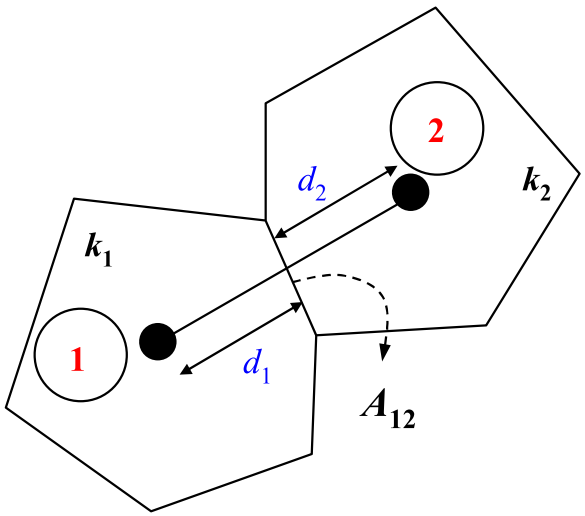
$$\Delta\Phi_{i\to e} = \Delta p_{i\to e} + \gamma \Delta Z - \Delta p_{c}$$

Upstream Mobility

$$M_{i\to e} = \frac{k_{r\,upstream}}{B_{i\to e}\,\mu_{i\to e}}$$

$$k_{rupstream} = \begin{cases} k_{ri} & \text{if } \Phi_i > \Phi_e \\ \dfrac{k_{ri} + k_{re}}{2} & \text{if } \Phi_i = \Phi_e \\ k_{re} & \text{if } \Phi_i < \Phi_e \end{cases}$$

Transmissibility



$$T_{12} = \frac{\alpha_1 \ \alpha_2}{\alpha_1 + \alpha_2}$$

$$\alpha_1 = \frac{k_1 \ A_{1-2}}{d_1}$$

$$\alpha_2 = \frac{k_2 \ A_{2-1}}{d_2}$$

A_{1-2} and A_{2-1} are the area of interface to the flow from $cell_1$ to $cell_2$ and $cell_2$ to $cell_1$ respectively. d is the distance from the control volume to the area of interface.

FVF & Viscosity

- Formation Volume Factor

$$B_{i\to e} = \frac{B_i + B_e}{2}$$

- Viscosity:

$$\mu_{i\to e} = \frac{\mu_i + \mu_e}{2}$$

Material Balance….

- The Material Balance of an evaluated cell, e is:

$$\Delta a_{i\to e}^{m}\Delta\Phi_{i\to e}^{n+1}=\frac{1}{\Delta t}\left[\left(\frac{V_pS}{B}\right)_e^{(n+1)}-\left(\frac{V_pS}{B}\right)_e^{n}\right]$$

$$\Delta a_{i\to e}^{m}\Delta\Phi_{i\to e}^{n+1}=\sum_{i=1}^{N}T_{i\to e}\,M_{i\to e}^{n}\Delta\Phi_{i\to e}^{n+1}$$

$$\sum_{e=1}^{Ncells}\left\{\sum_{i=1}^{N}T_{i\to e}\,M_{i\to e}^{n}\Delta\Phi_{i\to e}^{n+1}=\frac{1}{\Delta t}\left[\left(\frac{V_pS}{B}\right)_e^{(n+1)}-\left(\frac{V_pS}{B}\right)_e^{n}\right]\right\}$$

- Explicit:

$$\sum_{e=1}^{Ncells}\left\{\sum_{i=1}^{N} T_{i\to e}\; M_{i\to e}^{n}\Delta\Phi_{i\to e}^{n+1} = \frac{1}{\Delta t}\left[\left(\frac{V_p S}{B}\right)_e^{(n+1)} - \left(\frac{V_p S}{B}\right)_e^{n}\right]\right\}$$

- Implicit:

$$\sum_{e=1}^{Ncells}\left\{\sum_{i=1}^{N} T_{i\to e}\; M_{i\to e}^{n+1}\Delta\Phi_{i\to e}^{n+1} = \frac{1}{\Delta t}\left[\left(\frac{V_p S}{B}\right)_e^{(n+1)} - \left(\frac{V_p S}{B}\right)_e^{n}\right]\right\}$$

Well Modeling

- The Pressure at wellblock is not the same as The wellbore pressure.
- The wellblock sizes are significantly larger than wellbore radius.
- The difference between wellblock & wellbore pressure is proportional to a coefficient, Productivity/Injectivity Index.

Peaceman's Well Model

- Peaceman found that the pressure calculated for a well block is the same as the flowing pressure at an equivalent radius, r_o.

- Peaceman defined r_o as "the radius at which the steady-state flowing pressure for the actual well is equal to the numerically calculated pressure for the well block

Peaceman's Well Model

$$p_{wf} - p(r) = \frac{q\mu}{2\pi} \ln\left(\frac{r_w}{r_o}\right)$$

where,

p_{wf}	=	bottomhole well flowing pressure
$p(r)$	=	pressure at grid block containing the well
q	=	production rate of well
r_w	=	wellbore radius

Peaceman's Model & Cartesian Grid

- Uniform & isotropic reservoir:

$$r_o = 0.28\,\Delta x$$

- Non-uniform, isotropic reservoir:

$$r_o = 0.14(\Delta x^2 + \Delta y^2)^{1/2}$$

- Non-uniform, isotropic reservoir:

$$r_o = \frac{0.28\left[\sqrt{k_y/k_x}\,(\Delta x)^2 + \sqrt{k_x/k_y}\,(\Delta y)^2\right]^{1/2}}{(k_y/k_x)^{1/4} + (k_x/k_y)^{1/4}}$$

Well Rates and Pressures

- Oil $$q_o = J_{model}\left(\frac{k_r}{\mu B}\right)_o \left(p_{i,j} - p_{wf}\right)$$
- Water $$q_w = J_{model}\left(\frac{k_r}{\mu B}\right)_w \left(p_{i,j} - p_{wf}\right)$$
- Gas $$q_g = J_{model}\left(\frac{k_r}{\mu B}\right)_g \left(p_{i,j} - p_{wf}\right) + R_s q_o$$

Note that the pressure of the well and the pressure of its grid block are <u>different</u>.

Well Rates and Pressures

where: $$J_{model} = \frac{q}{\Delta p}\frac{\mu B}{k_r}$$

$$= \frac{2\pi(0.00633)kh}{\ln(r_o/r_w)+s} \quad \left[\frac{cp \cdot rcf}{d \cdot psi}\right]$$

$$= \frac{2\pi(0.00113)kh}{\ln(r_o/r_w)+s} \quad \left[\frac{cp \cdot rb}{d \cdot psi}\right]$$

and r_o is the equivalent radius of the well block.

Effective Wellbore Radius

- The key is the definition of the effective wellbore radius, r_o.

- Peaceman derived an expression for r_o that assumes:
  - (pseudo-)steady state flow
  - homogeneous reservoir
  - square grid blocks
  - isolated wells
  - incompressible flow

Peaceman Well Index

- Peaceman's well index can be derived analytically:

$$p(r) = p_{wf} + \frac{q_w \mu}{2\pi kh}\left(\ln \frac{r}{r_w}\right)$$

Pressure in the reservoir in terms of the wellbore pressure.

$$p(r) = p_0 + \frac{q_w \mu}{2\pi kh}\left(\ln \frac{r}{r_o}\right)$$

Pressure in the reservoir in terms of the well-block pressure.

Geometry

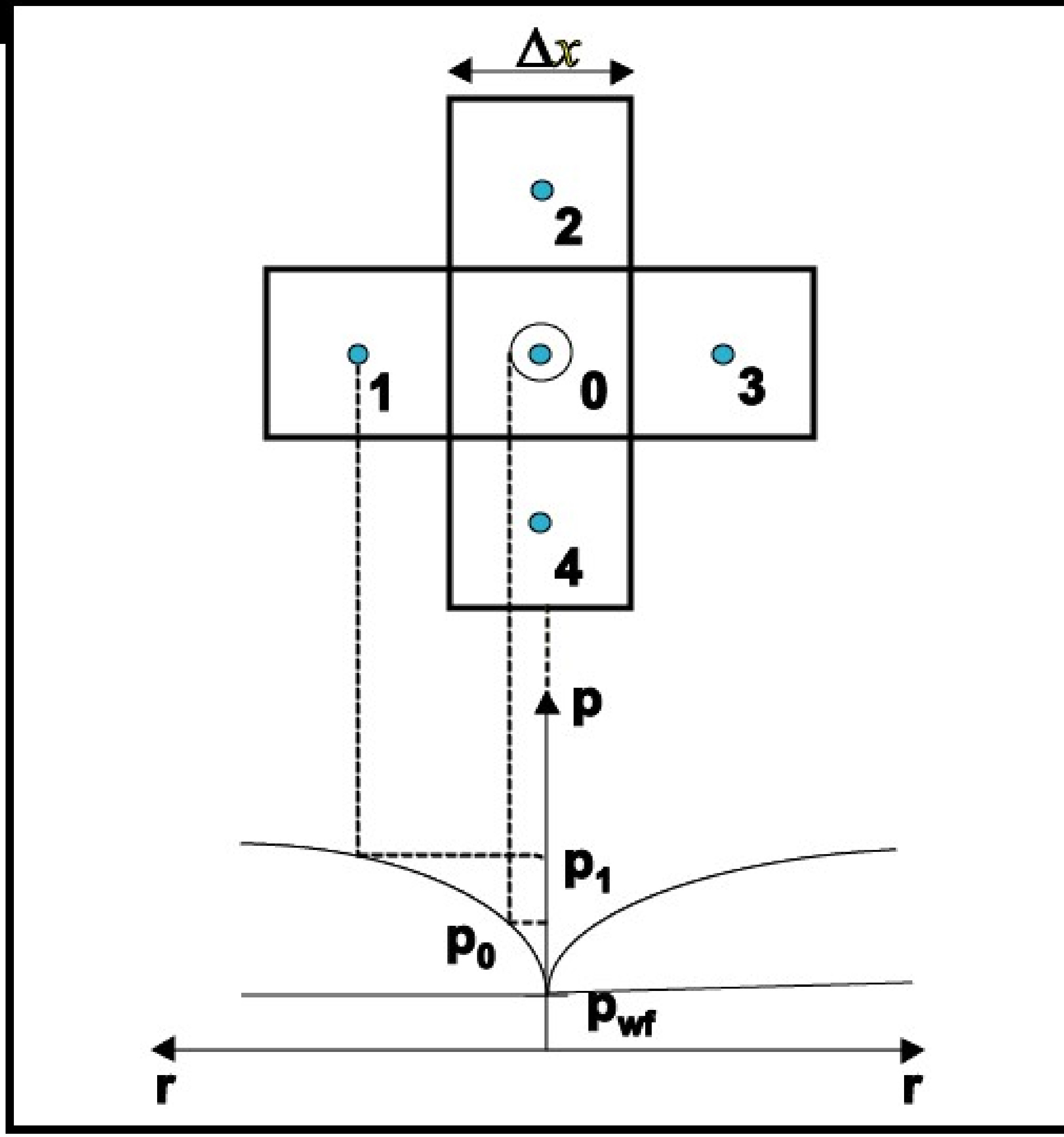


Figure: Tabiat Tan Yildiz, M.S. thesis

Peaceman Well Index

- These equations can be combined to give:

$$p_{wf} = p_0 + \frac{q_w \mu}{2\pi kh}\left(\ln \frac{r_w}{r_o}\right)$$

- Now the discrete form of the equations can be considered (block-to-block rates):

$$q_j = \frac{kA}{\mu}\left(\frac{p_j - p_0}{\Delta x}\right) \qquad j = 1,2,3,4$$

Peaceman Well Index

- Sum the flows into block zero:

$$q_w = \frac{kh}{\mu}\left[P_1 + P_2 + P_3 + P_4 - 4P_0\right]$$

1

- Now find the grid block pressures from the analytical solution:

$$p_i = p_0 + \frac{q_w \mu}{2\pi kh}\left(\ln \frac{\Delta x}{r_o}\right) \qquad i = 1,2,3,4$$

2

Peaceman Well Index

- Substituting 2 into 1 gives:

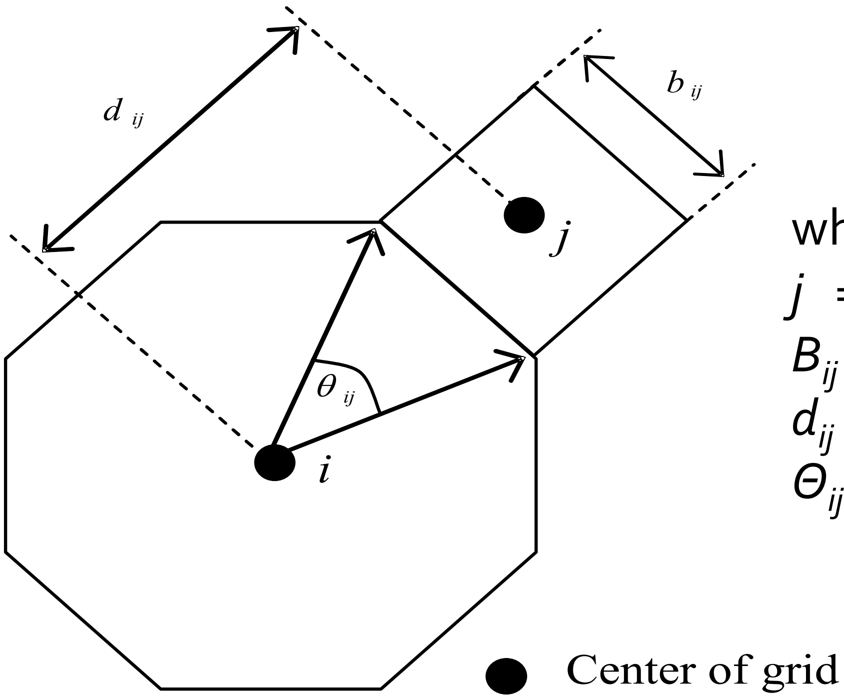
$$q_w = \frac{kh}{\mu}\left[4p_o + \frac{4q_w\mu}{2\pi kh}\ln\left(\frac{\Delta x}{r_o}\right) - 4p_o\right]$$

$$\Rightarrow \quad 1 = \frac{2}{\pi}\ln\left(\frac{\Delta x}{r_o}\right)$$

$$\Rightarrow \quad \exp\left(\frac{\pi}{2}\right) = \frac{\Delta x}{r_o}$$

$$\Rightarrow \quad \frac{r_o}{\Delta x} = \exp\left(-\frac{\pi}{2}\right) = 0.208$$

Modified Peaceman's Model



$$r_o = \exp\left(\frac{\sum_j \left(\frac{b}{d}\right)_{ij} \ln(d_{ij}) - \theta_{ij}}{\sum_j \left(\frac{b}{d}\right)_{ij}} \right)$$

where,

j = grid block that is neighbor of well block i

B_{ij} = length of side of polygon

d_{ij} = distance between the centers of grid block i and j

Θ_{ij} = angle open to flow ($\vartheta = 2\pi$ for an internal well, i.e. well located in the center of the block)

Polygon Wellblocks

- Equal sides:

$$\left(b/d\right)_{ij} = \tan\left(\pi/N\right)$$

$$r_o = d_{ij}\exp\left(\frac{-2\pi}{N\tan\left(\pi/N\right)}\right)$$

Well Index

- Well Index formula:

$$WI = \frac{2\,\pi\,k\,h}{\ln(\frac{r_o}{r_w}) + S}$$

$$P_{wf} = p(r) - \frac{q}{\lambda\,WI}$$

λ is the fluid mobility ($\lambda = k_r/\mu$) .

Well Constraints

- Producers are operated by the following constraints:
  - Constant rate of any phase
  - Constant bottomhole pressure
  - Constant drawdown

Producer Constraints

- Constrant Production Rate:

$$P_{wf}^{\;n+1} = p(r)^{n+1} + \frac{q_\alpha}{W\lambda_\alpha^{\;n+1}}$$

- Constant bottomhole pressure:

$$q_\alpha^{n+1} = WI\lambda_\alpha^{n+1}\left(p(r)^{n+1} - p_{wf}\right)$$

- Constrant drawdown:

$$p(r)^{n+1} - P_{wf}^{\;n+1} = -\frac{q_\alpha}{W\lambda_\alpha^{\;n+1}}$$

Injector Constraints

- Constant Injection Rate:

$$P_{wf} = p(r)^{n+1} + \frac{q_\alpha}{W\lambda_\alpha^{\;n+1}}$$

- Constant Injection Pressure:

$$q_\alpha = J\lambda_\alpha(P_{wf} - P_i)$$

Solution Technique

- Residual functions will be used to construct series of linear equations to solve the general material balance equation.

$$r_{o,g,w}^{n+1} = \sum_{e=1}^{Ncells} \left\{ \sum_{i=1}^{N} T_{i \to e} \; M_{i \to e}^{n+1} \Delta\Phi_{i \to e}^{n+1} - \frac{1}{\Delta t} \left[\left(\frac{V_p S}{B} \right)_e^{(n+1)} - \left(\frac{V_p S}{B} \right)_e^{n} \right] + q_{sc} \right\}$$

- For black oil model, the equation should be expanded into all forms of residual functions of oil, gas as water phase.

Residual Functions

- Oil Phase:

$$r_o^{n+1} = \sum_{e=1}^{Ncells} \left\{ \sum_{i=1}^{N} T_{i\to e} \, M_{o\,i\to e}^{\,n+1} \Delta\Phi_{o\,i\to e}^{\,n+1} - \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)_e^{(n+1)} - \left(\frac{V_p S_o}{B_o}\right)_e^{n} \right] + q_{osc} \right\}$$

- Gas Phase:

$$r_g^{n+1} = \sum_{e=1}^{Ncells} \left\{ \sum_{i=1}^{N} T_{i\to e} \, M_{g\,i\to e}^{\,n+1} \Delta\Phi_{g\,i\to e}^{\,n+1} - \frac{1}{\Delta t}\left[\left(\frac{V_p S_g}{B_g}\right)^{n+1} + R_{so}^{\,n+1}\left(\frac{V_p S_o}{B_o}\right)^{n+1} + R_{sw}^{\,n+1}\left(\frac{V_p S_w}{B_w}\right)^{n+1} - \ldots \right]\right\}$$

$$\left[\ldots \left(\frac{V_p S_G}{B_G}\right)_e^{n} - R_{so}^{\,n}\left(\frac{V_p S_o}{B_o}\right)^{n} - R_{sw}^{\,n}\left(\frac{V_p S_w}{B_w}\right)^{n} \right] + q_{g\,sc} + R_{so} q_{osc} + R_{sw} q_{wsc}$$

- Water Phase:

$$r_w^{n+1} = \sum_{e=1}^{Ncells} \left\{ \sum_{i=1}^{N} T_{i\to e} \, M_{wi\to e}^{\,n+1} \Delta\Phi_{w\,i\to e}^{\,n+1} - \frac{1}{\Delta t}\left[\left(\frac{V_p S_w}{B_w}\right)_e^{(n+1)} - \left(\frac{V_p S_w}{B_w}\right)_e^{n} \right] + q_{wsc} \right\}$$

Capillary Terms

$$P_{cow} = P_o - P_w \qquad \therefore P_w = P_o - P_{cow}$$

$$P_{cog} = P_g - P_o \qquad \therefore P_g = P_{cog} + P_o$$

Additional Equation???

Solving Equations:

- Using the residual function:

$$r_o, r_g, r_w = f(p_o, s_w, s_g)$$

  - Oil:

    $r_o = OIP^{n+1} - (OIP^n + Q_o)$

  - Gas:

    $r_g = GIP^{n+1} - (GIP^n + Q_g)$

  - Water:

    $r_w = WIP^{n+1} - (WIP^n + Q_w)$

Solving Equations:

- Using the residual function:
  - Oil:

$$r_o = OIP^{n+1} - (OIP^n + Q_o)$$

  - Gas:

$$r_g = GIP^{n+1} - (GIP^n + Q_g)$$

  - Water:

$$r_w = WIP^{n+1} - (WIP^n + Q_w)$$

Objective Function

- The objective functions are to satisfy $r_o = 0$, $r_g = 0$ and $r_w = 0$.
- We can consider our problem similar to a root-finding problem.
- Use any iterative method to solve all the equations.

Jacobian Matrix

- In order to solve the equations iteratively, we should compute the Jacobian matrix, *J* of those equations.

$$J = \begin{bmatrix} \frac{\partial r_o}{\partial p} & \frac{\partial r_o}{\partial s_w} & \frac{\partial r_o}{\partial s_g} \\ \frac{\partial r_g}{\partial p} & \frac{\partial r_g}{\partial s_w} & \frac{\partial r_g}{\partial s_g} \\ \frac{\partial r_w}{\partial p} & \frac{\partial r_w}{\partial s_w} & \frac{\partial r_w}{\partial s_g} \end{bmatrix}$$

Iterative Method

- The overall Newton-Raphson and Quasi-Newton iteration schemes form a linear equation of:

$$J^k\,[\Delta m]^{k+1} = -[r]^k$$

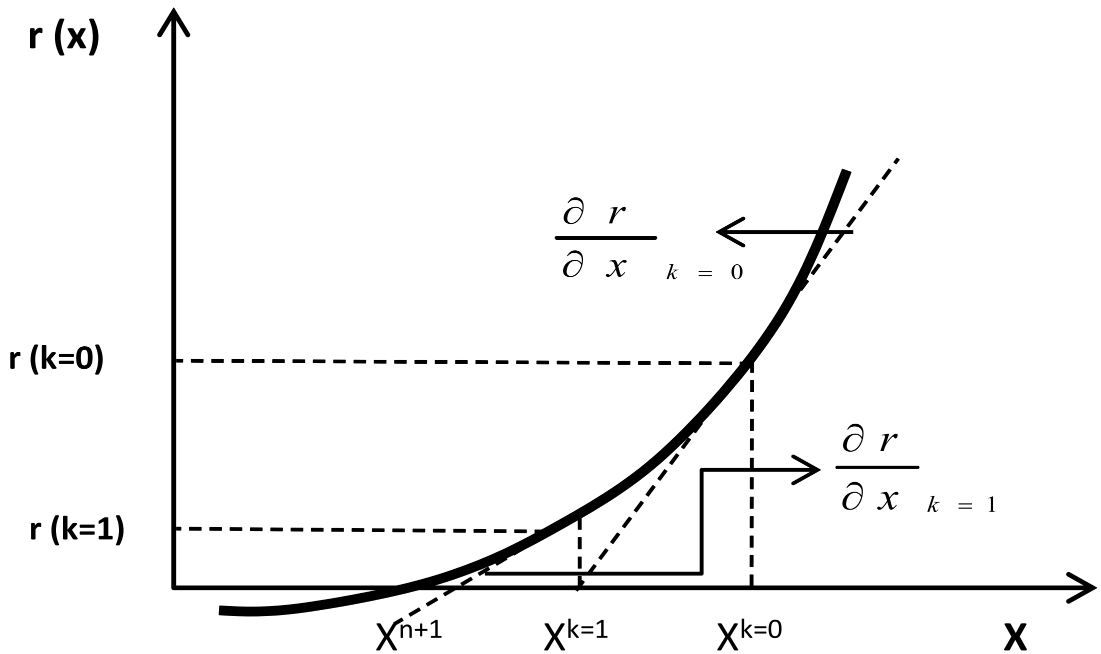
- The superscript of k indicates the iteration k^{th} and $\Delta m = \begin{bmatrix} dp \\ ds_w \\ ds_g \end{bmatrix}$

Iterative Method – Cont…

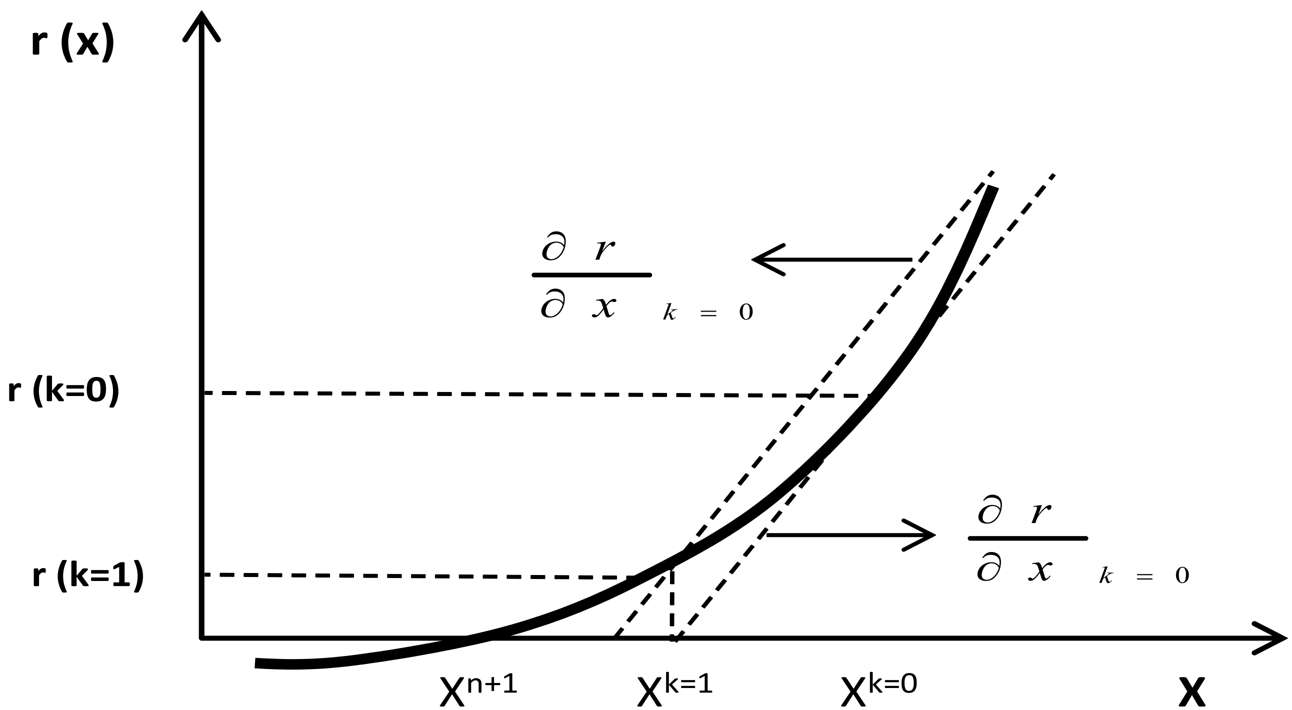
- p^{k+1}, $s_w{}^{k+1}$ and $s_g{}^{k+1}$ are updated using the following equation.

$$\begin{bmatrix} p \\ s_w \\ s_g \end{bmatrix}^{k+1} = \begin{bmatrix} p \\ s_w \\ s_g \end{bmatrix}^{k} - \begin{bmatrix} dp \\ ds_w \\ ds_g \end{bmatrix}^{k+1}$$

Newton Iterative Method



Quasi-Newton Iterative Method



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IMPES Method

Finite Difference Equations

$$a_{oE} = \left(\frac{Kr}{B\mu}\right)_{oE}\left(\frac{0.00633kh\Delta y}{\Delta x}\right)_E$$

- Oil

$$\Delta\left(a_o^n \Delta p^{n+1}\right) = \frac{1}{\Delta t}\left[\left(\frac{V_p S_o}{B_o}\right)^{n+1} - \left(\frac{V_p S_o}{B_o}\right)^n\right]$$

- Water

$$\Delta\left(\mathrm{a}_\mathrm{w}^\mathrm{n} \Delta \mathrm{p}^{\mathrm{n}+1}\right) = \frac{1}{\Delta \mathrm{t}}\left[\left(\frac{\mathrm{V_p S_w}}{\mathrm{B_w}}\right)^{\mathrm{n}+1} - \left(\frac{\mathrm{V_p S_w}}{\mathrm{B_w}}\right)^\mathrm{n}\right]$$

- Gas

$$\Delta\left(a_g^n \Delta p^{n+1}\right) = \frac{1}{\Delta t}\left[\left(\frac{V_p S_g}{B_g}\right)^{n+1} - \left(\frac{V_p S_g}{B_g}\right)^n\right]$$

IMPES Steps…

1) Calculate coefficients of the pressure equation
2) Calculate solution of the pressure equation implicitly (matrix equation) for: p^{n+1}
3) Calculate solution of the saturation equations explicitly for: S_o^{n+1}, S_w^{n+1}, and S_g^{n+1}

IMPES Method

- **4 unknowns per block:**

  p^{n+1}, S_o^{n+1}, S_w^{n+1}, and S_g^{n+1}

- **To find the unknowns, we need one more equation per block:** $S_o^{n+1} + S_w^{n+1} + S_g^{n+1} = 1$
- **Assures fluid volumes fit the pore volume**

IMPES Method

- Oil

$$S_o^{n+1} = \frac{B_o^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_o}{B_o}\right)^n + \Delta t \cdot \Delta\left(a_o^n \Delta p^{n+1}\right)\right]$$

- Water

$$S_w^{n+1} = \frac{B_w^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_w}{B_w}\right)^n + \Delta t \cdot \Delta\left(a_w^n \Delta p^{n+1}\right)\right]$$

- Gas

$$S_g^{n+1} = \frac{B_g^{n+1}}{V_p^{n+1}}\left[\left(\frac{V_p S_g}{Bg}\right)^n + \Delta t \cdot \Delta\left(a_g^n \Delta p^{n+1}\right)\right]$$

IMPES Method

Summing up all saturation equations:

$$\sum_{m=o,w,g} B_m^{n+1} \Delta\left(a_m^n \Delta p^{n+1}\right) =$$

$$\frac{1}{\Delta t}\left[V_p^{n+1} - \sum_{m=o,w,g} B_m^{n+1}\left(\frac{V_p S_m}{B_m}\right)^n\right]$$

IMPES Method

Approximate Vp^{n+1} on the right hand side using the identity:

$$V_p^{n+1} = V_p^n + \frac{\Delta V_p}{\Delta p}\left(p_i^{n+1} - p_i^n\right)$$

and a chord slope:

$$\frac{\Delta V_p}{\Delta p} = \frac{V_p^{n+1} - V_p^n}{p_i^{n+1} - p_i^n}$$

IMPES Method

Then,

$$V_p^{n+1} = V_p^n \left[1 + \frac{1}{V_p^n} \frac{\Delta V_p}{\Delta p} \left(p_i^{n+1} - p_i^n \right) \right]$$

$$= V_p^n \left[1 + c_f \left(p_i^{n+1} - p_i^n \right) \right]$$

where,

$$c_f \equiv \frac{1}{\phi} \frac{\Delta \phi}{\Delta p} = \frac{1}{V_p^n} \frac{\Delta V_p}{\Delta p}$$

Likewise,

$$B_m^{n+1} = B_m^n \left[1 - c_m \left(p_i^{n+1} - p_i^n \right) \right] \ (m = o, w, g)$$

IMPES Method

Final equation can now be written as:

$$\Delta\left(a_t^n \Delta p^{n+1}\right) = \frac{V_p^n c_t}{\Delta t}\left(p_i^{n+1} - p_i^n\right)$$

where :

$$\Delta a_t = \sum_{m=o,w,g} B_m^{n+1} \Delta a_m$$

$$c_t = c_f + \sum_{m=o,w,g} c_m S_m^n$$