SPE 145468

A Variable Rate Solution to the Nonlinear Diffusivity Gas Equation

Abelardo B. Barreto Jr., SPE, Petrobras; Alvaro M. M. Peres, SPE, Petrobras; Adolfo P. Pires, SPE, North Fluminense State University

This paper was prepared for presentation at the SPE Annual Technical Conference and Exhibition held in Denver, Colorado, USA, 30 October–2 November 2011.

Abstract
The hydraulic diffusivity equation that governs the flow of compressible fluids in porous media is nonlinear. Although gas well test analysis by means of the pseudo pressure function has become a standard field practice, the effect of viscosity and gas compressibility variation with pressure is often neglected. Moreover, in field operation, the gas well is submitted to a variable rate production in order to determine well/reservoir properties and an estimation of the absolute open flow (AOF). For slightly compressible fluids, variable rate can be properly handled by superposition in time. Unfortunately superposition cannot be offhand justified to gas reservoir due to its non-linear behavior. In this work a general solution which properly accounts for both fluid property behavior and variable rate is presented. The proposed solution, which is based on Green's Functions method by considering the viscosity-compressibility product change as a non-linear source term, can handle variable gas rate for several well and reservoir geometries of practical interest. From the general solution, an analytical expression for variable rate tests of a fully penetrating vertical well in an infinite gas reservoir is derived. This expression is applied to a synthetic data set to calculate the pressure response for a buildup test in an infinite homogeneous reservoir. The results compared to a finite difference numerical simulator shows close agreement. It is also shown that the dimensionless pseudo pressure converges to the slightly compressible fluid solution for long shut-in times. Thus, at such long times, Horner analysis and log-log derivative plot can be applied to obtain good estimative of reservoir parameters as already discussed previously in literature.

Introduction
Single phase flow of real gas in a homogeneous isotropic porous media is governed by the hydraulic diffusivity equation:

$$\nabla \cdot \left(\frac{p}{\mu Z} \nabla p \right) = \frac{\phi \mu c_t}{k} \frac{p}{\mu Z} \frac{\partial p}{\partial t}. \tag{1}$$

This equation is non-linear due to the pressure dependence of gas properties (that is, viscosity, gas compressibility and real gas deviation Z factor). The basis of most theoretical work is the linearization of this partial differential. Al-Hussainy, Ramey and Crawford (1966) proposed the pseudo pressure transformation in order to replace the pressure as the dependent variable. This transformation is defined by

$$m(p) = 2 \int_{p_b}^{p} \frac{p'}{\mu(p')\, Z(p')} dp', \tag{2}$$

where p_b is a reference pressure. Upon this transformation, Eq. 1 becomes similar to the slightly compressibility fluid flow equation:

$$\nabla^2 m = \frac{1}{\eta(m)} \frac{\partial m}{\partial t}. \tag{3}$$

The diffusivity η is originally a function of pressure, that is $\eta(p)=k/[\phi\mu(p)c_t(p)]$. However, as *m(p)* is a one-to-one function of *p*, the diffusivity can be written as a function of *m(p)* as well so Eq. 1 can be rewritten in terms of the pseudo pressure as dependent variable only. Note that Eq. 3 remains non-linear as the diffusivity group η is pseudo pressure dependent meaning that the transformation *m(p)* does not linearize the gas flow equation completely.

Some attempts were made to account for the pressure dependence on η. An approximate solution from a perturbation technique was derived for radial flow for constant rate by Kale and Mattar (1980). Even though some unnecessary and unsound simplifications were made in their derivation, some of the conclusions regarding the solution behavior are indeed correct. Peres, Serra and Reynolds (1990a, 1990b) developed an exact solution under constant rate production for a vertical gas well in an infinite reservoir using the Boltzmann self-similar transformation. A perturbative solution for the same problem was also presented, showing that the solution truncated at the second order term is a very accurate approximation and can be applied for engineering purposes. Their approach, however, can be applied on self-similar problems only which severely restrict the applicability of the technique to solve other nonlinear well testing problems of practical interest.

In field operations, well tests are designed and run with variable rate for several reasons. This is especially critical for gas reservoirs. For slightly compressible fluids (liquids), variable rate is handled by Duhamel's Principle which is routinely applied to well test problems (Thompson and Reynolds - 1986). As the gas diffusivity equation remains non-linear even after the use of the pseudo pressure, superposition in time cannot be applied, that is, a variable rate solution cannot be obtained by superimposing the constant rate solution. Some articles apply the superposition theory to nonlinear problems by assuming the diffusivity term η constant (Gupta and Andsage - 1967). If this condition is fulfilled, at least in an approximated sense, variable rate interpretation techniques derived for slightly compressible fluids can be extended to gas well tests (Samaniego and Cinco-Ley - 1991). Recently, von Schroeter and Gringarten (2007) also investigated the superposition principle applied to non-linear gas problems. However, the problem is simplified again by considering beforehand the diffusivity term η as a constant.

This brief review shows that although some theoretical work for constant rate gas well testing has been done, there is a lack in the petroleum engineering literature of a more fundamental and rigorous attempt to solve the variable rate case. This gap is addressed in this paper. In the following, a general 3-D variable source strength solution for gas reservoirs is presented. Then, the solution for a vertical well produced by variable rate is obtained from the general solution. An analytical equation for the buildup period for gas wells is derived next and then compared to the results of a finite difference simulator.

Mathematical Model and Background

For a homogeneous isotropic reservoir of infinite extent in both *x* and *y* directions, with no fluid flow across the top and bottom of a formation with uniform thickness, the isothermal flow of a real gas (neglecting non-Darcy flow, gravitational effects and compositional changes) is described by the following system of governing equation coupled with proper initial and boundary conditions:

$$\nabla^2 m_D - f_D(x_D, y_D, z_D, t_D) = \eta_D \frac{\partial m_D}{\partial t_D} \tag{4}$$

$$m_D(x_D, y_D, z_D, t_D = 0) = 0 \tag{5}$$

$$\begin{cases} \lim\limits_{x_D, y_D \to \pm\infty} m_D(x_D, y_D, z_D, t_D) = 0 \\ \left(\dfrac{\partial m_D}{\partial z_D}\right)_{z_D = bottom} = 0 \\ \left(\dfrac{\partial m_D}{\partial z_D}\right)_{z_D = top} = 0 \end{cases} \tag{6}$$

The dimensionless variables for pseudo pressure, time and diffusivity that appears in Eqs. 4-6 are defined respectively by $m_D = \dfrac{\pi k h}{q_{ref}} \dfrac{T_{sc}}{p_{sc} T_R} \Delta m(p)$, $t_D = \dfrac{k t}{\phi(\mu c_t)_i l_c^2}$ and $\eta_D = \dfrac{\mu c_t}{(\mu c_t)_i}$. The pseudo pressure change $\Delta m(p)$ is given by $\Delta m(p) = m(p_i) - m(p)$, whereas q_{ref} and l_c represent, respectively, a convenient reference gas rate and a characteristic length. The spatial variables *x*, *y* and *z* are made dimensionless by this characteristic length. The term f_D represents dimensionless point sources/sinks defined by $f_D(x_D, y_D, z_D, t_D) = 2\pi \dfrac{\tilde{q}_{sc}(x,y,z,t)}{q_{ref}} l_c^2\, h$, where $\tilde{q}_{sc}$ represents the volumetric gas rate by unit volume at position *(x, y, z)* at a given time *t* at standard conditions. For gas injection, we define $\tilde{q}_{sc} > 0$, thus f_D is positive. Consequently, for gas production from the point *(x, y, z)* the flow rate $\tilde{q}_{sc} < 0$ and f_D is negative. The above formulation and its solution can handle single or multi-wells problems with variable rates as well as different wellbore geometries (such as hydraulic fractured wells, restricted entry-wells and horizontal wells). By a standard coordinate transformation, Eq. 4 can be modified to handle permeability anisotropy. In such case, it is assumed that directions *x*, *y* and *z* are aligned to the principal permeability axes.

A general solution to Eqs. 4-6 is given by (Barreto Jr. - 2011),

$$m_D(x_D,y_D,z_D,t_D) = -\int_\Gamma\int_0^{t_D}\left\{ f_D(x_D',y_D',z_D',t_D') + w(m_D)\frac{\partial m_D}{\partial t_D'}(x_D',y_D',z_D',t_D')\right\}\times \tag{7}$$
$$\times G_D(x_D,x_D',y_D,y_D',z_D,z_D',t_D,t_D')\, dt_D'\, dx_D'\, dy_D'\, dz_D'$$

where $w = (\eta_D - 1)$ gives the relative variation of the viscosity-compressibility product from its initial value. In Eq. 7 G_D denotes the dimensionless Green's Function. Green's Functions for several geometries and boundary conditions can be found in Carslaw and Jaeger (1959) and Beck *et all* (1992). The first integral sign represents a multidimensional integration over the spatial domain. Eq. 7 is an integral-differential equation of Volterra type that is an equivalent representation of the formulation given by Eqs. 4-6 (it can be shown that Eq. 7 indeed satisfies Eqs. 4-6; see Barreto Jr. - 2011). Existence and uniqueness proofs to an equivalent non-linear heat-conduction problem leading to a solution similar to Eq. 7 can be found in Koval'chuk and Lopushanskaya (1993).

Because of the group $w(\partial m_D/\partial t_D)$ in the integrand in Eq. 7, the dimensionless pseudo-pressure m_D solution given by Eq. 7 is implicit and it can only be computed by an iterative method or by an asymptotic expansion of m_D. To start the iteration process, make $m_D(x_D,y_D,z_D,t_D) = p_D(x_D,y_D,z_D,t_D)$, where p_D represents the mathematical solution for a slightly compressible fluid, that is, the solution for the case where η is constant at its initial value. Thus, at the end of the first iteration Eq. 7 becomes

$$m_D(x_D,y_D,z_D,t_D) = -\int_\Gamma\int_0^{t_D}\left\{ f_D(x_D',y_D',z_D',t_D') + w(p_D)\frac{\partial p_D}{\partial t_D'}(x_D',y_D',z_D',t_D')\right\}\times \tag{8}$$
$$\times G_D(x_D,x_D',y_D,y_D',z_D,z_D',t_D,t_D')\, dt_D'\, dx_D'\, dy_D'\, dz_D'$$

Barreto Jr. (2011) shows that Eq. 8 is equal to the expression obtained by an asymptotic expansion technique (perturbation) truncated at the first order term. Barreto Jr. (2011) has also shown that for a few typical well tests problems, Eq. 8 provides excellent results when compared to a finite difference numerical simulator.

Computational Aspects. Besides the dependence of *w* upon m_D, Eq. 8 represents a multidimensional integral differential equation which has to be solved numerically. In this work the multidimensional numerical integration package CUBA (Hahn - 2005a, 2005b) was used to this end. This library provides four integration algorithms, and three of them, called Vegas, Suave and Divonne are Monte Carlo methods, and the Cuhre algorithm is a deterministic one. A comparison test showed that the Vegas algorithm gives more accurate and stable results for the problems considered here. A computer program was written to provide numerical solutions for the nonlinear gas equation for different geometries.

Variable Rate Solution for a Vertical Well

In this section we consider the case of a single and fully penetrating vertical well producing at variable sandface gas flow rate $q_{sc}(t)$ measured at standard conditions. In addition to the simplifying hypothesis given before, it is assumed here that there is no wellbore storage effect and the formation is not damaged (i.e, skin factor is zero). For this case, the cylindrical coordinates are preferable (Fig. 1). For the sake of completeness, the mathematical formulation is given below.

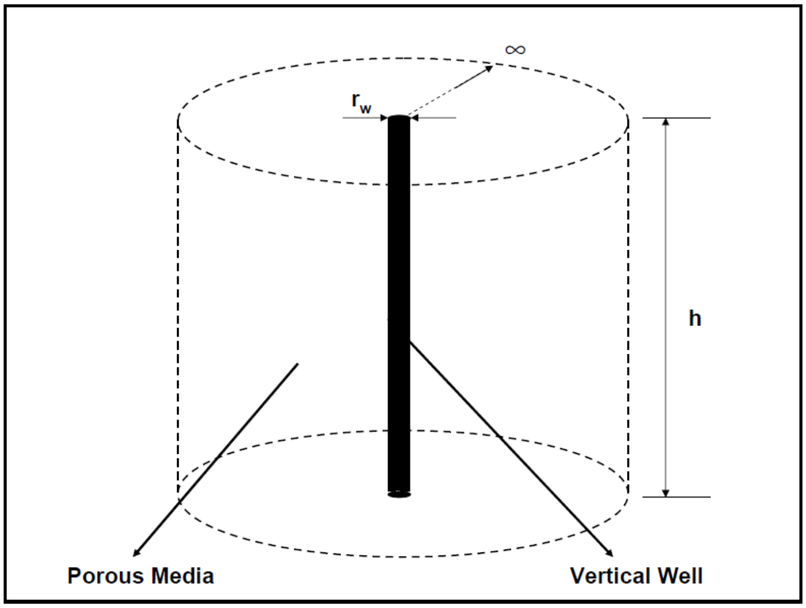

Figure 1 – Radial Geometry

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_{DV}}{\partial r_D}\right)-f_D(r_D,t_D)=\eta_D\frac{\partial m_{DV}}{\partial t_D} \tag{9}$$

$$m_{DV}(r_D,t_D=0)=0 \tag{10}$$

$$\lim_{r_D\to\infty} m_{DV}(r_D,t_D)=0\,. \tag{11}$$

The dimensionless variables are the same defined previously, with $l_c = r_w$, where r_w stands for the wellbore radius. Thus, the dimensionless radius is defined by $r_D = r/r_w$. The subscript V was introduced here to avoid confusion between the pseudo pressure solution for variable gas rate case and the constant rate solution. The solution for Eq. 9-11 can be obtained directly from Eq. 7 by changing the coordinate system fom Cartesian to plane radial, which gives

$$m_{DV}(r_D,t_D)=-\int_0^{\infty}\int_0^{t_D}\left\{f_D(r_D^{'},t_D^{'})+w(m_{DV})\frac{\partial m_{DV}}{\partial t_D^{'}}(r_D^{'},t_D^{'})\right\}\times G_D(r_D,r_D^{'},t_D,t_D^{'})\,dt_D^{'}\,dr_D^{'} \tag{12}$$

For a uniform flux cylindrical source/sink of radius r_0, the dimensionless term f_D is given by

$$f_D(r_D,t_D)=-2\pi\, q_{wD}(t_D)\delta(r_D-r_{D0}), \tag{12.a}$$

where q_{wD} denotes the dimensionless wellbore rate defined by $q_{wD}(t_D)=q_{sc}(t)/\,q_{ref}$ and δ is the Dirac's delta function. Substituting the source/sink term $f_D(r_D,t_D)$ in Eq. 12 and using Delta's function sifting property, one gets

$$\begin{aligned} m_{DV}(r_D,t_D) &= 2\pi\int_0^{t_D} q_{wD}(t_D^{'})G_D(r_D,r_{D0},t_D,t_D^{'})\,dt_D^{'} \\ &\quad -\int_0^{t_D}\int_0^{\infty} w(m_{DV})\frac{\partial m_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'} \end{aligned} \tag{13}$$

For the case of interest of a single fully penetrating well we make $r_{D0} = 0$ (so the cylindrical source becomes a line source as desired), then the solution becomes

$$m_{DV}(r_D,t_D) = 2\pi\int_0^{t_D} q_{wD}(t_D^{'})\; G_D(r_D,0,t_D,t_D^{'})\; dt_D^{'}$$
$$-\int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}} G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\, dt_D^{'} \tag{14}$$

where we have already made the approximation $m_{DV}(r_D,t_D) = p_{DV}(r_D,t_D)$ in the integrand of the second integral of Eq. 13 as discussed before. Note that $p_{DV}(r_D,t_D)$ denotes the mathematical solution for a slightly compressible fluid under the same flow rate history. The dimensionless Green's functions in Eqs. 13 and 14 are (Carslaw and Jaeger - 1959; Beck *et all* - 1992)

$$G_D(r_D,r_{D0},t_D,t_{D0}) = \frac{1}{4\pi(t_D - t_{D0})}\exp\left(-\frac{r_D^2 + r_{D0}^2}{4(t_D - t_{D0})}\right)\mathrm{I}_0\left(\frac{r_D\, r_{D0}}{2(t_D - t_{D0})}\right) \tag{15.a}$$

$$G_D(r_D,0,t_D,t_{D0}) = \frac{1}{4\pi(t_D - t_{D0})}\exp\left(-\frac{r_D^2}{4(t_D - t_{D0})}\right) \tag{15.b}$$

The function I_0 in Eq. 15.a is the modified Bessel's function of the first kind and order zero (Abramowitz and Stegun - 1972). Note that if the diffusivity remains constant we have $w = 0$ and the second integral in Eqs. 14 vanishes. Thus the first integral in Eq. 14 represents the variable rate solution for slightly compressible fluids, that is

$$p_{DV}(r_D,t_D) = 2\pi\int_0^{t_D} q_{wD}(t_D^{'})\; G_D(r_D,0,t_D,t_D^{'})\; dt_D^{'} \tag{16}$$

By noting that $p_{DC}(r_D,t_D) = 2\pi\int_0^{t_D} G_D(r_D,0,t_D,t_D^{'})\; dt_D^{'}$ where p_{DC} represents the mathematical solution for constat rate ($q_{wD} = 1$) for slightly compressible fluids, Eq. 16 can also be written as

$$p_{DV}(r_D,t_D) = \int_0^{t_D} q_{wD}(t_D^{'})\frac{\partial p_{DC}(r_D,t_D - t_D^{'})}{\partial t_D^{'}}\; dt_D^{'} \tag{17}$$

which is the familiar Duhamel's principle. So, the general variable rate solution can be expressed as follows

$$m_{DV}(r_D,t_D) = p_{DV}(r_D,t_D) - \int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}} G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\, dt_D^{'} \tag{18}$$

Eq. 18 indicates that the pseudo pressure solution for a real gas under variable rate production is given by the corresponding variable rate solution for slightly compressible fluids plus a correction term which accounts for the departure of the viscosity-total compressibility product from its initial value.

Validation – The Buildup Test Case

Ins this section, the variable rate solution given by Eq. 18 will be applied to a classical buildup test (Fig. 2), when the well is produced with a constant gas rate during a certain period of time t_p (the drawdown period) after which the well is closed (buildup period) and the wellbore pressure increases. The solution derived for this particular case will be compared to the results of a finite difference simulator.

The gas flow rate schedule for a buildup test is

$$q_{wD}(t_D) = \begin{cases} 1, & t_D < t_{pD} \\ 0, & t_D > t_{pD} \end{cases} \tag{19}$$

Hence, using Eq. 19 in the first integral of Eq. 18 for $t_D > t_{pD}$ yields

$$m_{DV}(r_D,t_D)=p_{DC}(r_D,t_D)-p_{DC}(r_D,t_D-t_{pD}) -\int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}\,G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'} \tag{20}$$

The buildup pseudo pressure response can be expressed in terms of the dimensionless shut-in time, Δt_D. With $t_D = t_{pD} + \Delta t_D$, Eq. 20 turns to

$$m_{DV}(r_D,t_D)=p_{DC}(r_D,t_{pD}+\Delta t_D)-p_{DC}(r_D,\Delta t_D) -\int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}\,G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'} \tag{21}$$

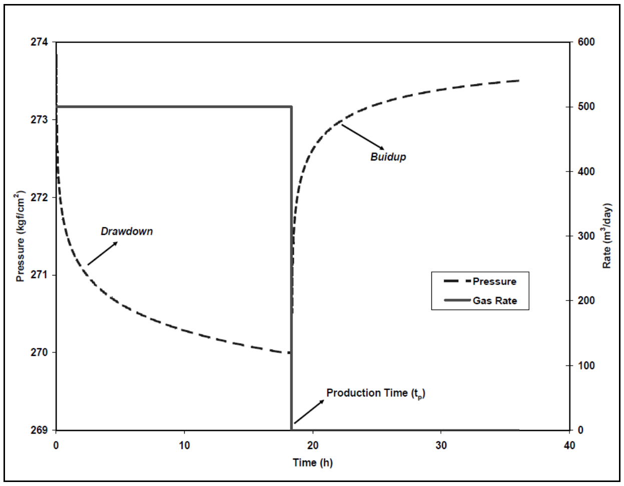

Figure 2 – Drawdown and Buildup Test Gas Rate Schedule

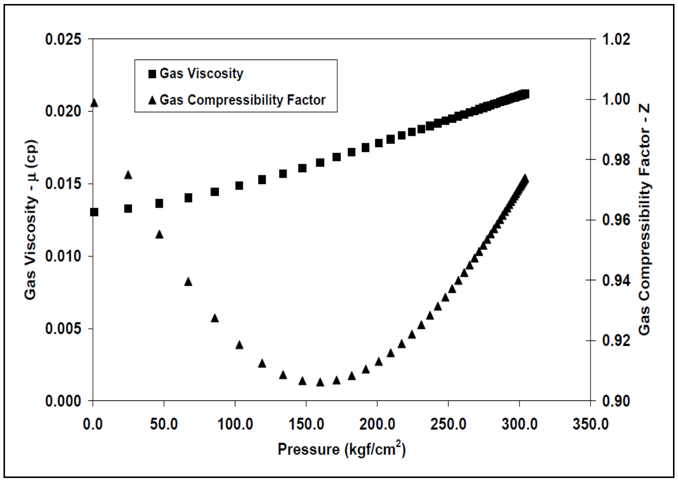

Figure 3 – Gas Properties at Reservoir Temperature

The first two terms in the right hand side of Eq. 21 represent the classical buildup solution for slightly compressible fluids. The third term can be seen as the deviation of the gas pseudo pressure solution (m_{DV}) from the liquid solution (p_{DV}). From a computational point of view it seems interesting to split the integral in Eq. 21 into two terms: one from 0 to t_{pD} (which accounts for the w variation during the drawdown) and the other from t_{pD} to t_D (which accounts for w variation that occurs during the buildup period). That is,

$$\int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}\,G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'}=\int_0^{t_{pD}}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}\,G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'} +\int_{t_{pD}}^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D^{'},t_D^{'})}{\partial t_D^{'}}\,G_D(r_D,r_D^{'},t_D,t_D^{'})dr_D^{'}\,dt_D^{'} \tag{22}$$

Eq. 22 clearly shows that the diffusivity deviation factor w is associated to a specific test period.

In order to test the formulation proposed in this paper and applied to a vertical gas well test in an infinite reservoir producing with variable rate, a synthetic data set was created. Gas properties are shown in Figure 3. Reservoir and other fluid properties are given in Table 1. The pseudo pressure versus pressure curve for the gas properties shown in Figure 3 is presented on Figure 4. The diffusivity deviation factor w variation with respect to the pseudo pressure $m(p)$ is shown in Figure 5.

Property	Value	Unit
k	1.0	mD
ϕ	0.1	---
c_f	0.000050	$(kgf/cm^2)^{-1}$
h	40	m
c_{gi}	0.002513	$(kgf/cm^2)^{-1}$

μ_i	0.021086	cp
p_i	300	Kgf/cm^2
p_{sc}	1.03323	Kgf/cm^2
T_{sc}	15.6	^{o}C
T_R	60.0	^{o}C
q_{sc}	400000	std m^3/day
t	15	h

Table 1 – Rock and Fluid Properties

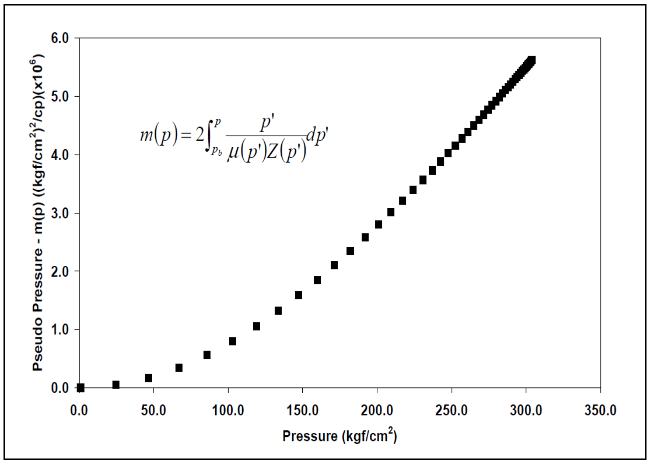

Figure 4 - Pseudo Pressure x Pressure Behavior

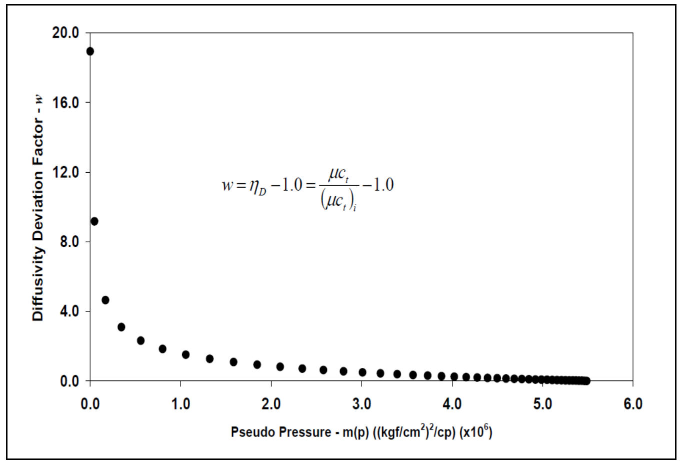

Figure 5 - Diffusivity Deviation Factor*w* x *m(p)* Behavior

A commercial finite difference simulator was used to compare the results obtained by Eq. 18 for this specific dataset. A radial grid was built to represent a vertical well response in the finite difference simulator. The numerical grid consisted of 200 cells in the r direction. The external radius of the reservoir was set large enough to avoid boundary effects during the simulated well test period. As usual, the radial cells were logarithmicly distributed in order to capture the higher pressure gradients near the well and to minimize truncation errors. Vertical flow and gravity were made negligible by using a single layer model. A very fine timestep scheme was also used in the simulator to ensure better accuracy. The simulator used solves for pressure so its results were converted to pseudo pressure by the relationship show in Fig. 4.

The comparison between the variable rate solution proposed in this work and the numerical simulation results will be done at the wellbore radius $r = r_w$, that is, at $r_D = 1$. This is the most interesting case for field applications and it is also the position where the effect of the non-linear behavior is stronger. Then, for $r_D = 1$, Eq. 21 becomes

$$m_{wDV}(t_D) = p_{wDC}(t_{pD} + \Delta t_D) - p_{wDC}(\Delta t_D) + m_{wDV1}(t_D) \tag{23}$$

where

$$m_{wDV1}(t_D) = -\int_0^{t_D}\int_0^{\infty} w(p_{DV})\frac{\partial p_{DV}(r_D', t_D')}{\partial t_D'} G_D(1, r_D', t_D, t_D')dr_D'\, dt_D' \tag{24}$$

Fig. 6 shows the log-log plot of the dimensionless pseudo pressure m_{wDV} and the dimensionless pseudo pressure derivative m_{wDV}' for the drawdown period. The agreement is excellent for both m_{wDV} and m_{wDV}' solutions, especially for dimensionless times greater than 10. Fig. 7 compares the first order pseudo pressure m_{wDV1} obtained by the solution presented in this work (Eq. 24) and the m_{wDV1} obtained from finite difference simulator results. The simulator data points follow the general behavior displayed by our solution. It is clear from this figure that the finite difference m_{D1} is less accurate than the ones obtained by Eq. 24.

The same analysis is done for the buildup period. Fig. 8 shows the log-log plot of the dimensionless pseudo pressure $\widetilde{m}_{wDV}$ and the dimensionless pseudo pressure derivative $\widetilde{m}_{wDV}{}'$. Close agreement is found again for both $\widetilde{m}_{wDV}$ and $\widetilde{m}_{wDV}{}'$ solutions, mainly for dimensionless times greater than 10. Fig. 9 shows the log-log plot for the same data shown in Fig. 8 comparing the first order pseudo pressure m_{wDV1} obtained by the solution presented in this work (Eq. 24) to the m_{wDV1}

obtained from finite difference simulator results. Again, the simulation results present a general behavior similar to our solution. It seems from this figure that the finite difference m_{wDV1} is less accurate than the one calculated by Eq. 24.

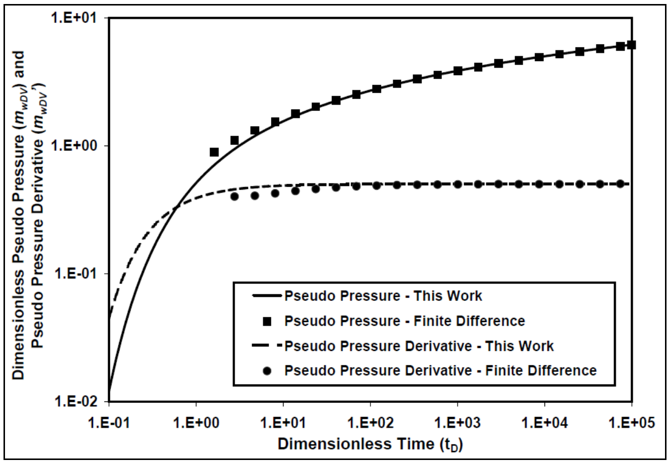

Figure 6 – Dimensionless Pseudo Pressure and Pseudo Pressure Derivative - This Work versus Finite Difference Simulation - Drawdown

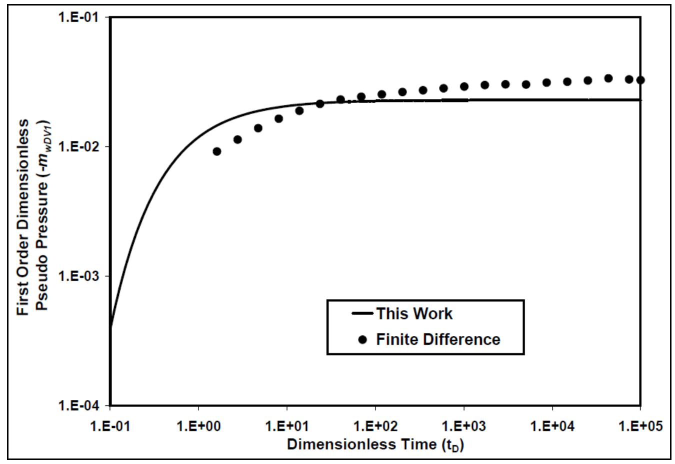

Figure 7 – First Order Pseudo Pressure - This Work and Finite Difference Simulation - Drawdown

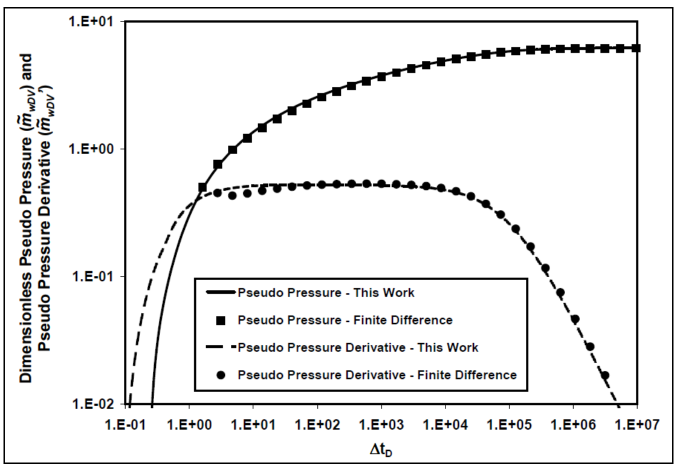

Figure 8 – Dimensionless Pseudo Pressure and Pseudo Pressure Derivative - This Work versus Finite Difference Simulation - Buildup

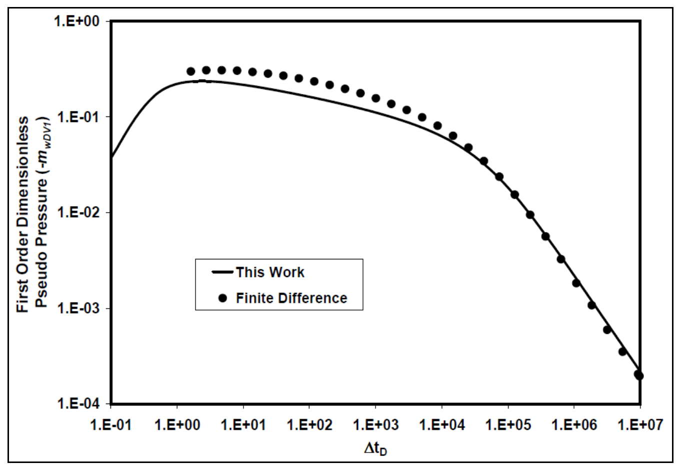

Figure 9 – First Order Pseudo Pressure - This Work and Finite Difference Simulation - Buildup

Horner Method is a classical technique for buildup test data analysis (Lee, Rollins and Spivey - 2003). In this method, the wellbore pressure (for liquids) or wellbore pseudo pressure (for gases) data points are plotted against Horner's time function $(t_p + \Delta t)/\Delta t$. The Horner plot in dimensionless variables for our example is shown in Fig. 10. In this figure, curves for $\tilde{m}_{wDV}(\Delta t_D)$ for the gas, and $\tilde{p}_{wDV}(\Delta t_D)$ for the liquid are compared to the numerical simulation pseudo pressure results. These dimensionless variables are calculated, respectively, by

$$\tilde{m}_{wDV}(\Delta t_D) = m_{wDV}(t_{pD}) - m_{wDV}(t_{pD} + \Delta t_D) \tag{25}$$

$$\begin{aligned}\tilde{p}_{wDV}(\Delta t_D) &= p_{wDV}(t_{pD}) - p_{wDV}(t_{pD} + \Delta t_D) \\ &= p_{wDC}(t_{pD}) - p_{wDC}(t_{pD} + \Delta t_D) + p_{wDC}(\Delta t_D)\end{aligned} \tag{26}$$

Fig. 10 shows that, for long times, the pseudo pressure behavior is close to the liquid solution. Hence, it is possible to conclude that classical interpretation methods for slightly compressible fluids can also be used for buildup gas wells if the buildup period is long enough. Fig. 11 shows the log-log plot for the same data shown in Fig. 10 comparing the first order pseudo pressure m_{wDV1} obtained by the solution presented in this work to the m_{wDV1} obtained from finite difference simulator results. Again, the simulation results present a general behavior similar to our solution, including the abnormal behavior close to the shut in time.

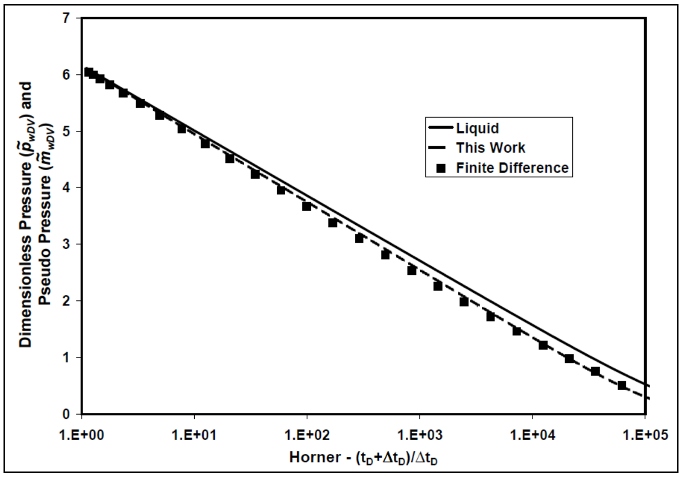

Figure 10 – Horner Plot

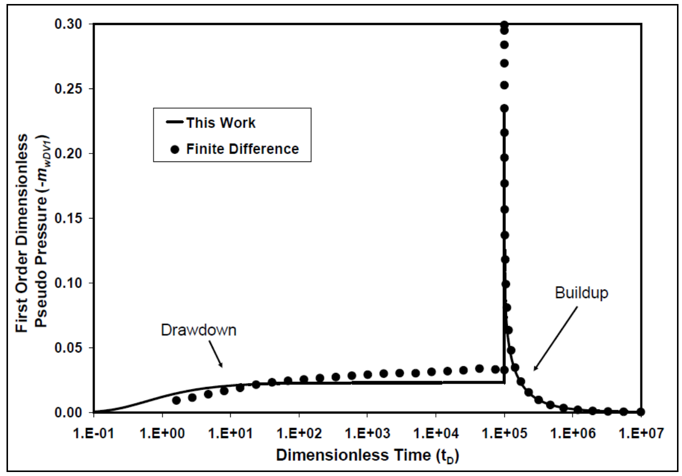

Figure 11 - First Order Pseudo Pressure - This Work and Finite Difference Simulation – Drawdown and Buildup Periods

Another interesting issue of this solution is the rate dependence results. Peres, Serra and Reynolds (1990) showed that, for drawndown period, the first order dimensionless pseudo pressure (m_{wDV1}) is rate dependent. In order to evaluate this behavior for buildup period, the dimensionless pseudo pressure was determined from the same reservoir data for two different rates: 200M and 400M std m^3/day. The log-log plot of the drawdown dimensionless pseudo pressure m_{wDV} and its logtime derivative m_{wDV}' for both rates are shown in Fig. 12. In Fig. 13 the drawdown first order dimensionless pseudo pressure m_{wDV1} is compared. Figs. 14 and 15 show the same results for the buildup period. It is interesting to observe that the rate dependence on pseudo pressure behavior occurs, mainly, in the first order term.

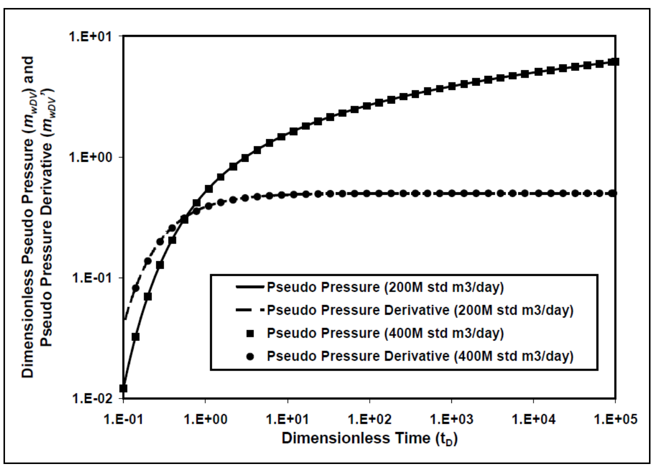

Figure 12 - Dimensionless Pseudo Pressure and Pseudo Pressure Derivative - Drawdown

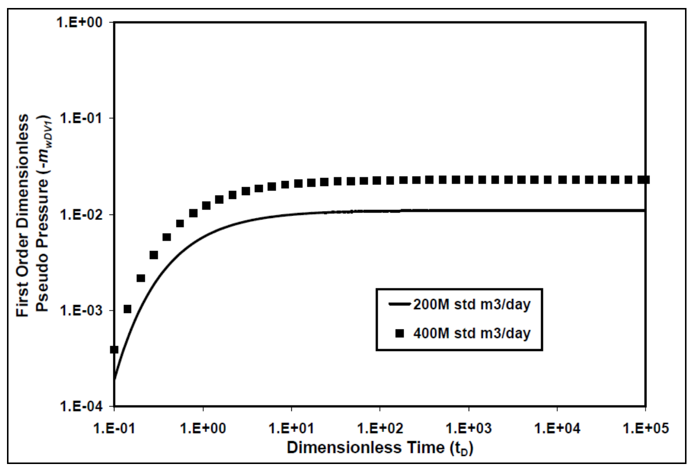

Figure 13 - First Order Pseudo Pressure - Drawdown

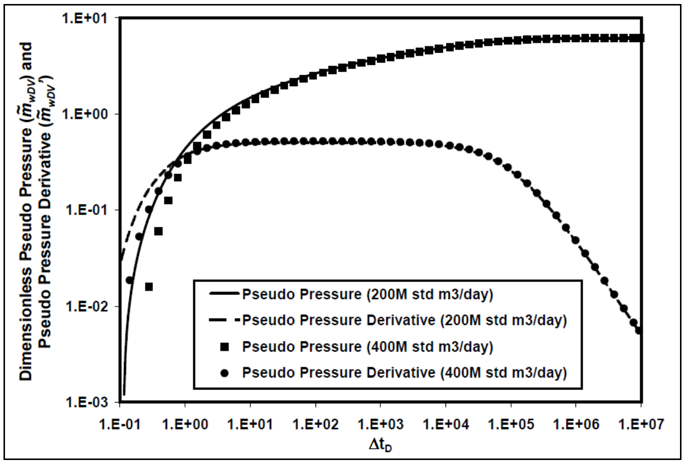

Figure 14 - Dimensionless Pseudo Pressure and Pseudo Pressure Derivative - Buildup

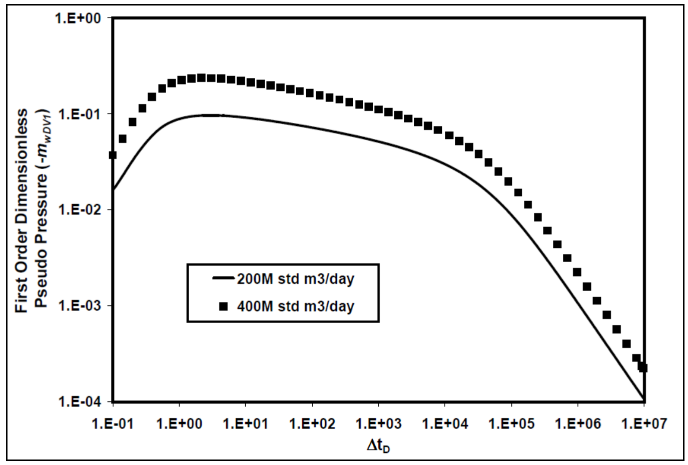

Figure 15 - First Order Pseudo Pressure - Buildup

Conclusions

In this work, a general and new solution to the Nonlinear Gas Diffusivity Equation is presented, and the variable gas rate solution is derived. The results for a base case are compared to finite difference simulation with excellent agreement. The pseudo pressure response shows the same general behavior observed for slightly compressible fluid solution. So, classical interpretation techniques can be apllied to variable gas rate tests. The results presented here show that the pseudo pressure m_{wDV} solution is rate dependent in both drawdown and buildup periods.

Nomenclature

c_f = Formation compressibility, $(kgf/cm^2)^{-1}$
c_g = Gas compressibility, $(kgf/cm^2)^{-1}$
$c_t = c_f + c_g$; Total compressibility, $(kgf/cm^2)^{-1}$
f_D = Dimensionless source/sink term
G_D = Dimensionless Green function
h = Reservoir thickness, m
k = Absolute permeability, mD
$m(p)$ = pseudo pressure, $(kgf/cm^2)^2/cp$
$\Delta m(p)$ = pseudo pressure drop, $(kgf/cm^2)^2/cp$
m_D = Dimensionless pseudo pressure
m_{D1} = First order dimensionless pseudo pressure
m_D' = Dimensionless pseudo pressure derivative
m_{DV} = Variable rate dimensionless pseudo pressure
m_{wDV} = Variable rate dimensionless wellbore pseudo pressure
$\tilde{m}_{wDV}$ = Variable rate dimensionless pseudo pressure change
m_{DV1} = Variable rate first order dimensionless pseudo pressure
m_{wDV1} = Variable rate first order dimensionless wellbore pseudo pressure
m_{wDV}' = Variable rate dimensionless wellbore pseudo pressure derivative
$\tilde{m}_{wDV}'$ = Variable rate dimensionless pseudo pressure change derivative
l_C = characteristic length, m
p = Pressure, kgf/cm^2
p_b = Reference pressure, kgf/cm^2
p_i = Initial reservoir pressure, kgf/cm^2
p_{sc} = Pressure in standard conditions, kgf/cm^2
p_{wf} = Well-flow pressure, kgf/cm^2
p_D = Dimensionless pressure solution for a slightly compressible fluid
p_{DC} = Constant rate dimensionless pressure solution for a slightly compressible fluid
p_{wDC} = Constant rate dimensionless wellbore pressure solution for a slightly compressible fluid
p_{DV} = Variable rate dimensionless pressure solution for a slightly compressible fluid
p_{wDV} = Variable rate dimensionless wellbore pressure solution for a slightly compressible fluid
$\tilde{p}_{wDV}$ = Variable rate dimensionless wellbore pressure change

q_{sc} = Gas production rate in standard conditions, std m^3/day
r = Radial distance, m
r_D = Dimensionless radial distance
r_w = Wellbore radius, m
t = time, hours
Δt = elapsed shut-in time, hours
Δt_D = dimensionless shut-in time
t_D = Dimensionless time
t_p = producing time, hours
t_{pD} = dimensionless producing time
T_R = Reservoir temperature, ^oC
T_{sc} = Standard Temperature condition, ^oC
w = Dimensionless diffusivity deviation
Z = Gas compressibility factor
ρ = Gas density, kg/m^3
ϕ = Porosity
μ = Gas viscosity, cp
δ = Dirac's delta function
η = Hydraulic diffusivity, mD (kgf/cm^2)/cp
η_D = Dimensionless diffusivity
Γ = Spatial domain

Acknowledgements
The authors would like to thank Petrobras for financial support and SPE Macae Section for OnePetro subscription. We also like to thank José Roberto Rodrigues from Petrobras/CENPES for calling our attention to Koval'chuk and Lopushanskaya paper.

References
Abramowitz, M. and Stegun, I. A., Handbook of Mathematical Functions. New York: Dover Publications, 1972.

Al-Hussainy, R., Ramey Jr., H. J. and Crawford, P. B., 1966, The flow of real gases through porous media, Journal of Petroleum Technology, SPE Paper 1243A, vol. 18, pp. 624-636.

Barreto Jr., A. B. 2011. *Non-linear Gas Diffusivity Equation Solution by Green's Functions*. PhD Thesis (in Portuguese). Universidade Estadual Norte Fluminense, Macaé, Brazil

Beck, J. V., Cole, K. D., Haji-Sheikh, A., and Litkouhi, B., 1992, Heat Conduction Using Green's Functions. Philadelphia: Hemisphere Publishing Corporation.

Bourdet, D., 2002, Well Test Analysis: The Use of Advanced Interpretation Models, Elsevier Sciences.

Carslaw, H. S. and Jaeger, J. C., 1959, Conduction of Heat in Solids. New York: Oxford Science Publications.

Gupta, K. C., and Andsage, R. L., Application of variable rate analysis technique to gas wells, Fall Meeting of the Society of Petroleum Engineers of AIME, New Orleans, Louisiana, 1967, SPE Paper 1836.

Hahn, T., 2005a, Cuba - a library for multidimensional numerical integration, Computer Physics Communications, no. 168, pp. 78-95.

Hahn, T., 2005b, Cuba - a library for multidimensional numerical integration, report, Max-Planck-Institut für Physik. Available for download at http://cdsweb.cern.ch/record/728672.

Kale, D. and Mattar, L., 1980, Solution of a non-linear gas flow equation by the perturbation technique, JCPT, PETSOC 80-04-06, pp. 63-67.

Koval'chuk, B. V. and Lopushanskaya, G. P., 1993. The Green's function method in stationary and nonstationary nonlinear heat-conduction problems, Journal of Mathematical Sciences **66** (6), pp 2587-2591.

Lee, W. J., Rollins, J. B. and Spivey, J. P., Pressure Transient Testing. Richardson: Society of Petroleum Engineers Inc., 2003.

Peres, A. M. M., Serra, K. V., and Reynolds, A. C., 1990a, Toward a unified theory of well testing for nonlinear-radial-flow problems with application to interference tests, SPE Formation and Evaluation, SPE Paper 18113, pp. 151-160.

Peres, A. M. M., Serra, K. V., and Reynolds, A. C., 1990b, Supplement to SPE 18113, Toward a unified theory of well testing for nonlinear-radial-flow problems with application to interference tests, SPE Formation and Evaluation, SPE Paper 20402.

Samaniego, F., and Cinco-Ley, H., Transient pressure analysis for variable rate testing of gas wells, Low Permeability Reservoirs Symposium, Denver, Colorado, 1991, SPE Paper 21831.

Thompson, L., and Reynolds, A., Analysis of variable-rate well-test pressure data using Duhamel's principle, SPE Formation Evaluation, vol. 1, pp. 453-469, October 1986, SPE Paper 13080.

von Schroeter, T. and Gringarten, A. C., 2007, Superposition principle and reciprocity for pressure transient analysis of data from interfering wells, SPE Journal, SPE Paper 110465.