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Diffusivity Equation for Real Liquids with Gas in Solution

E. Lanfranchi, S.A. Consultores C.S.C.

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ABSTRACT

A rather comprehensive theory has been developed to describe fluid flow phenomena in porous media. This theory, under the constraints of appropriate simplifying assumptions, ultimately leads to expressions which enable the petroleum engineer to characterize reservoir behaviour. In this context, one frequently treats some form of the diffusivity equation in conjunction with data obtained from well tests to arrive at specific reservoir parameters. For example, so-called pressure transient analysis can be used to arrive at estimates of the formation thickness-permeability product, the storage capacity, the average reservoir pressure, etc. For liquid flow, the technique involves reducing the diffusivity equation to a form where pressure becomes the independent variable. This requires the assumptions that the liquid is both, ideal and slightly compressible.

In this work it has been developed an alternative approach which does not depend upon the assumptions cited above. An additional intent is to investigate the effects of these suppositions on computed pressure distributions in a one-dimensional system.

References and ilustrations at end of paper

INTRODUCTION

The mathematical description of fluid flow through porous media has been known for a long time. However, in the past most solutions to reservoir problems were limited to simple cases where the applications did not involve much computation. The development of computers and new numerical techniques[1,2], have made it possible to treat the more difficult problems of petroleum engineering.

In 1934, Hurst[3] and Muskat[4] introduced mathematical equations describing flow of a single-phase slightly-compressible fluid through a homogeneous porous medium. They presented several solutions for some relatively simple cases. The firts effort to apply these equations to build-up pressure analysis for incompresible fluid flow was made by Muskat[5]. The advance in methods and computers allowed the study of more complicated situations, but in all of them, it is implicitly assumed that the liquid phases in a reservoir are ideal and that they furthermore are slightly compressible. When a large amount of gas is disolved in a crude under reservoir conditions, the departure from ideality and slight compressibility may be substantial. The present work was undertaken to evaluate the effects of these assumptions and develop an alternative approach which does not rely upon them. To this end, we confine our attention to a comparison of computed pressure distributions in a one-dimensional system.

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Density Equation Development

The oil density equation in this work is derived from the PVT curves of oil formation volume factor and solution gas-oil ratio as functions of pressure. We first assume the existence of some pressure below the bubble point, say P_1, such that the PVT curves for pressures $P \geq P_1$ can be essenciallity represented by a linear relationship. Thus the pertinent curves will appear like those in Fig. 1.

We can therefore write

$$B_o = B_{ob} + m(P - P_b) \quad (1)$$

$$R_s = R_{sb} + n'(P - P_b) \quad (2)$$

for $P \geq P_1$ where m and n' are the appropriate slopes. Above the bubble point the slope m is negative while below it is positive. Similarly n' is positive for $P < P_b$ and zero for $P > P_b$. As shown in Appendix A, these relationships can be used to arrive at the following expression for oil density

$$\rho = \frac{\rho_{ob} B_{ob} + n(P - P_b)\rho_g}{B_{ob} + m(P - P_b)} \quad (3)$$

where $n = n'/5.614$ and ρ_g is the gas density at standard conditions. It will be observed that Eq. (3) reduces to the well-know expression

$$\rho = \frac{\rho_{ob} B_{ob}}{B_o} \quad \text{when } P > P_b$$

Formulation of the Flow Equation

The equation for single phase fluid flow in a porous medium in obtained by combining the continuity equation and Darcy's Law. The former is an expression of conservation of mass in terms of a fluid velocity vector. Darcy's Law, on the other hand, expresses the macroscopic velocity in terms of the potential gradient at a point in the system. Thus we get

$$\nabla\{\rho \frac{k}{\mu}(\nabla P + \rho G \nabla h)\} - G = -\phi \frac{\partial \rho}{\partial t} \quad (4)$$

It we restrict our attention to $P < P_b$ then the accumulation term on the right-hand side of Eq. (4) will carry a minus sign. In this region we will employ the equation of state given by Eq. (3). Our interest is not centered on pressures in excess of the bubble point since for $P > P_b$ the assumption of slightly compressible fluids is probably valid.

Eq. (3) can be employed in Eq. (4) to eliminate the pressure (details of this development are in Appendix B). Thus,

$$\nabla\{\rho \frac{k}{\mu}[\frac{B_{ob}(n\rho_g - m\rho_{ob})}{(n\rho_g - m\rho)^2}\nabla\rho + \rho g \nabla h]\} - G = -\phi \frac{\partial \rho}{\partial t} \quad (5)$$

If permeability and viscosity are considered constant then

$$\nabla\{\rho[\frac{\nabla\rho}{(n\rho_g - m\rho)^2} + \frac{\rho_g \nabla h}{B_{ob}(n\rho_g - m\rho_{ob})}]\} - \frac{\mu G}{k B_{ob}(n\rho_g - m\rho_{ob})} = \frac{\phi\mu}{k B_{ob}(n\rho_g - m\rho_{ob})}\frac{\partial \rho}{\partial t} \quad (6)$$

If no sources are present and only one-dimensional flow is considered, Eq. (6) becomes

$$\frac{\partial}{\partial x}[\frac{\rho}{(n\rho_g - m\rho)^2}\frac{\partial \rho}{\partial x}] = \frac{\phi\mu}{B_{ob} k (n\rho_g - m\rho_{ob})}\frac{\partial \rho}{\partial t} \quad (7)$$

Furthermore if we define the groups,

$$X = \frac{X}{L}$$

$$T = \frac{t\, k\, B_{ob}(n\rho_g - m\rho_{ob})}{\phi\, \mu\, \alpha\, L^2}$$

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Where α is a conversion factor then Eq. (7) becomes

$$\frac{\partial}{\partial X}\left[\frac{p}{(np_g - mp)^2}\frac{\partial p}{\partial X}\right] = \frac{\partial p}{\partial T} \qquad (8)$$

If the coefficient in the brackets of Eq. (8) is set equal to one, then the problem reduces to

$$\frac{\partial^2 p}{\partial X^2} = \frac{\partial p}{\partial T} \qquad (9)$$

where we have arbitrarily changed the sign on the right-hand side (this can be accomplished by redefining T). This expression is identical to the single-phase one-dimensional flow equation for an ideal liquid[5]. It should be emphasized that in Eq. (8) no assumptions of ideality have been made. Indeed, in those cases where a large amount of gas dissolves in the oil under reservoir conditions, assuming an ideal liquid may not be valid.

Numerical Solution

Eq. (8) is non-linear and dependent upon the density prescribed at a point in space and time. Because of this, a numerical procedure was required to solve the problem. This procedure in described in the following paragraphs. The same technique can be used to generate solutions to Eq. (9) by simply setting $p/(np_g - mp)^2 = 1$ and redefining T. A finite difference analog of Eq. (8) is readily constructed by superposing a network on the region of interest in the usual manner. In this instance equal grid spacing was employed and a Crank-Nicholson[6] scheme was used to arrive at the following finite difference equations:

$$1/2\ \{\ \Delta_x\ A^{n+1}{}_x\ \Delta_x\ p^{n+1} + \Delta_x\ A^{n}{}_x\ \Delta_x\ p^{n}\} = -\ \Delta_t\ p \qquad (10)$$

where

$$A = p_i\ /\ [(np_g - mp_i)h_x]^2$$

$$\Delta_x\ A_x\ \Delta_x p = A_{i+1/2}\ (p_{i+1}-p_i) - A_{i-1/2}\ (p_i-p_{i-1})$$

and

$$\Delta_t p = (p_i^{n+1} - p_i^{n})/h_t$$

Eq. (10) can also be expressed as

$$h_t\ A^{n+1}{}_{i-1/2}\ p^{n+1}{}_{i-1} + [\ 2 - h_t(A_{i-1/2} + A_{i+1/2})]^{n+1}\ p_i^{n+1} + h_t\ A^{n+1}{}_{i+1/2}\ p^{n+1}{}_{i+1} = \Psi^{n}_{i} \qquad (11)$$

for

$$i = 1, 2, \ldots\ldots, N$$

where

$$\Psi^{n}_{i} = p^{n}_{i} - h_t\ \Delta_x\ A^{n}_{x}\ \Delta_x\ p^{n}_{i}$$

This represents a set of linear algebraic equations which require simultaneous solution. Alternatively we can state this as the matrix problem $\beta p = \Psi$ where β is a tridiagonal coefficient matrix and Ψ is a known vector defined by the right-hand-side of Eq. (11). Since the coefficients are p-dependent a point Gauss-Seidel[6,7] iterative scheme is employed which permits the use of updated values. In matrix notation the problem then becomes

$$p^{(k+1)} = (D - L)^{-1}\ U\ p^{(k)} + (D-L)^{-1}\ \Psi \qquad (12)$$

where D, L and U are the diagonal, lower triangular and uppe triangular matrices respectively of the matrix.

Density distributions were determined by solving both Eqs. (8) and (9) using the method just outlined subject to the following conditions

(i) $p = p_{ob}$, $T = 0$, $0 \le X \le 1$
(ii) $p = p_{o1}$, $T > 0$, $X = 0$
(iii) $p = p_{ob}$, $T > 0$, $X = 1$

It will be recalled that Eq. (9) assumes the liquid is ideal where as Eq. (8) does not. Thus comparison of the density (or pressure)

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distributions will reflect the effect of this assumption. Obviously the deviation will be dependent on the amount of dissolved gas in a particular oil. PVT data were obtained on various crudes which were placed into three categories - those having a low, medium or high amount of gas in solution. Each category represents an increasing departure from an ideal liquid since ideality is more closely approached in dilute solutions. For convenience the low, medium and high ranges of gas in solution are designated as groups A, B and C respectively. Each group has the following arbitrary limits on Rs:

Group A:	0	$\leq$	$R_s \leq$ 1000 SCF/STB
Group B:	1000	$<$	$R_s \leq$ 2000 SCF/STB
Group C:			$R_s >$ 2000 SCF/STB

There were two crude samples in each group. Data for these are found in Table 1.

The linear region was assigned a lenght; L=5000 ft; porosity=.20 and permeability= 5. md. Calculations were carried out simulating time steps of 30 days. After time zero, the pressure at the near boundary inmediately decreases and the density reaches a value reflecting this change. At the far boundary the value of the density is maintained constant consistent with condition (iii). A value of A is firts computed for i = 1, 3, ..., N at time step n. Time is increased and the values of density at the previous time are used as the K_{th} iterates. New values of density at the $(k+1)^{st}$ iteration are calculated using Eq. (11).

Closure is achieved when

$$\max \left| \rho^{k}_{i} - \rho^{k+1}_{i} \right| < \delta$$

$$1 \leq i \leq N$$

for $\delta > 0$, an arbitrary tolerance. When this occurs the $(k+1)^{st}$ iterate is taken as the value of ρ^{n+1} and the procedure is repeated for the next time.

After the density profiles were obtained a corresponding pressure profile was constructed using Eq. (3) for the non-ideal case, and Eq. (13) Below for the ideal liquid case

$$\rho = \rho_{ob}\, e^{Co\,(P - Pb)} \qquad (13)$$

This expression is the integrated form of $C_o = 1/\rho\,(\partial \rho/\partial P)$ assuming constant compressibility. In many applications the further assumption that the liquid is slightly compressible is frequently invoked and one can write

$$\rho = \rho_{ob}\,[1 + C_o(P-P_b)] \qquad (14)$$

which is obtained by the firts two terms of the power serires expansion of the exponential function in Eq. (13). It is also of interest to investigate the deviation incurred by this supposition, consequently, pressure profiles were prepared employing Eq. (14).

Obviously, the deviations between the pressure profiles for the three cases will be dependent upon the value of C_o that enters into Eqs. (13) and (14). The correct value in each is that which satisfacies the near boundary condition, i.e. for the strictly ideal case

$$C_o = \frac{L_n\, \rho_{o1}/\rho_{ob}}{P_1 - P_b} \qquad (15)$$

and for the slightly-compressible case

$$C_o = \frac{\rho_{o1} - \rho_{ob}}{\rho_{ob}\,(P_1 - P_b)} \qquad (16)$$

Different deviations were determined by computing the following terms:

Absolute Point Deviation (lppc)

$$E_{APi,j} = P_{ki,j} - P_{ni,j} \qquad (17)$$

Absolute Time Step Deviation (lppc)

$$E_{AT\,j} = [\Sigma^{N}_{i=1}\,(P_{ki} - P_{ni})^2/N]^{1/2} \qquad (18)$$

Percentage Point Deviation (%)

$$E_{PPi,j} = 100\,(P_{ki,j} - P_{ni,j})/P_{ni,j} \qquad (19)$$

Percentage Time Step Deviaton (%)

$$E_{PTj} = 100\,[\Sigma_{i=1}{}^{N}\,((P_{ki} - P_{ni})/P_{ni})^2/N]^{1/2} \qquad (20)$$

Considering the difference between P_1 and P_b (AP) for each case, the normalized deviation was calculated using,

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Absolute Normalized Deviation (Dimesionless)

$$E_{ANj} = [\Sigma^{N}_{i=1} (P_{ki} - P_{ni})^2/N]^{1/2} /AP \quad (21)$$

Percentage Normalized Deviation (Dimensionless)

$$E_{PNj} = [\Sigma^{N}_{i=1} ((P_{ki} - P_{ni})/P_{ni}^2/N)^{1/2} 100/AP \quad (22)$$

Where:

P_n = Pressure corresponding to Eq (8)

P_k = Pressure corresponding to Eq (13) for ideal liquid or Eq (14) for Slightly compressible liquid

N = Number of observation points (11)

AP = $P_b - P_1$

i = Observation Point

j = Time Step

RESULTS

The results of this study are presented in terms of pressure rather than density distributions, since we are more frequently concerned with pressure behaviour. For each case mentioned, before are generated a set of Tables and Figures where all the deviations previously defined were analized.

All the cases show the same behaviour, consenquently it will be presented in details only Case 2 of Group A. The Absolute Point Deviation (E_{AP}) observed as funtion of time are presented in Figures 2 and 3 for ideal in slightly compressible fluid. In Figures 4 and 5 we can observed the Percentage Point Deviation (E_{PP}) as funtion of Time for ideal and slightly compressible fluid.

It will be noted that the assumption of ideality incurrs less deviation than that encountred when is further assumed that the liquid is slightly compressible. The deviation increase rapidly as the time increase until reachs the maximum value, after that the deviation decrease as the time increase. For all the cases the maximum deviation is not reached at the same time in all the points. The maximum deviation value and the time requiered to reach it increase as the gas is solution at P_b increase.

The Absolute Time Step Deviation (E_{AT}) is presented in Figure 6 and the Absolute Normalized Deviation in Figure 7 (E_{AN}). It will be noted that in both Figures the deviation reach a maximum at same time and after that point it decrease as the time increase. The same behaviour has been observed for the corresponding percentage deviation (Figures 8 and 9). When the Deviation is presented as a percentage, the behavior of the two cases is similar.

The Figures 10 and 11 depict the dependence of Absolute Normalized Deviation (E_{AN}) on Gas in Solution at different times for ideal and slightly compressible fluid. It will be noted that the assumption of ideality incurs less deviation than that encountered when one further assumes that the liquid is slightly compressible. In fact, at the later times (Figs. 10 and 11) the deviation is more than doubled for the slightly compressible case. Furthermore, the deviation increses as R_{sb} increases, as expected.

The time dependence of the normalized deviation is more crearly seen in Fig. 12. This plot was constructed by selecting representative values of R_{sb} for the low, medium and high amounts of gas in solution (i.e., Groups A, B and C) as indicated on the plot and then cross-plotting the data of all the Cases. In this figure we observe that the deviation passes through a maximum for each curve. Furthermore, the locus of the maxima shifts in the direction of increasing time as the gas solubility increases. The existence of the maxima reflects the fact that diffusion phenomena (as described by Eqs. (8) and (9)) pass through a transient stage into a quasi-steady state regime as time increases. The maxima in some sense reflect the time whem this transition takes place. To the left of the peaks we essentially have a transient condition while to the right we are approaching a steady state. The shift in the locus of the maxima to the rigth indicates that as the gas in solution increases the transition in delayed, i.e., a longer time in required to reach steady state.

These results may be of practical sinficance in performing pressure transient analyses. Since most well tests are conducted under transient flow conditions, the measurements can very well correspond to those times when the normalized deviation (in Fig. 12) is approaching a maximun. We would anticipate that the greatest deviation would occur in tight reservoirs containing a curde

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with high gas solubility. This in because a longer time is required to approach esteady state and the deviation in larger as R_s increases.

CONCLUSIONS

From this study we conclude the following for oil reservoirs below the bubble point pressure:

1.For one-dimensional problems the assumption of ideality for the oil can lead to a maximum Porcentaje Deviation for Points E_{PP}, in the computed pressure distribution as follows:

For R_s = 750 SCF/STB $E_{PP} \leq$ .65%
For R_s = 1500 SCF/STB $E_{PP} \leq$ 1.25%
For R_s = 3000 SCF/STB $E_{PP} \leq$ 2.20%

2.With the additional assumption that the oil is slightly compressible, these deviations become greater, i.e.:

For R_s = 750 SCF/STB $E_{PP} \leq$ 1.15%
For R_s = 1500 SCF/STB $E_{PP} \leq$ 2.40%
For R_s = 3000 SCF/STB $E_{PP} \leq$ 3.20%

3.The normalized deviation passes through a maximum which shifts in the direction of increasing time when the amount of gas in solution increases.

The results of this work suggest that the assumptions of ideality and slight compressibility may have significative effect in those cases where the early time flow capacity calculation are used.

NOMENCLATURE

B_o = Oil formation volume factor, RB/STB

B_{ob} = Oil formation volume factor at bubble point conditions, RB/STB

B_{os} = Oil formation volume factor at standard conditions, RB/STB

B_{o1} = Oil formation volume factor at P_1, RB/STB

C_o= Oil compressibility, Vol/Vol/psi

G= Source or sink term

g= Acceleration due to gravity, ft/sec^2

h_t= Dimensionless distance increment

h_x= Dimensionless time increment

k= Permability, md

L = Length of the system, ft

m = Slope of B_o-curve, RB/STB/psi

n = Slope of R_s-curve, SCF/STB/psi

p = Pressure, psia

P_b = Bubble-point (saturation) pressure, psia

P_1 = Pressure, psia

R_s = Solution gas-oil ratio, SCF/STB

R_{sb}= Solution gas-oil ratio at bubble point pressure, SCF/STB

R_{s1}= Solution gas-oil ratio, SCF/STB

T = Time group

t = Time, days

V = Velocity, cm/sec

X = Normalized distance

x = Distance, ft

α = Conversion factor to practical units, 158.06 cm^2, md, psi, day/ft^2, Darcy At, sec

Δ = Increment

δ = Tolerance

ϵ = Normalized error

μ = Viscosity, cp

ρ = Oil density, lb/CF

ρ_g = Gas density at standard conditions, lb/SCF

ρ_{ob}= Oil density at bubble point conditions, lb/CF

ρ_{or}= Oil density at reservoir condition, lb/CF

ρ_{os}= Oil density at s.c. lb/CF

ρ_{o1}= Oil density at pressure p_1, lb/CF

Θ = Potencial $\int_{p_o}^{p} dp/(\rho(p) + gh$

$\emptyset$ = Porosity

Subscripts and Superscripts

i= Position space index

j= Time index

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k= Iteration step index

n= Time step index

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APENDIX A

DERIVATION OF THE OIL DENSITY EQUATION

The oil density equation was derived from PVT curves of oil formation volume factor and solution gas-oil ratio as functions of pressure. In this derivation it is assumed that the following linear reltionships are valid for $p \geq p_1$:

$$B_o = B_{ob} + m\,(P - P_b) \qquad (A\text{-}1)$$

$$R_s = R_{sb} + n'(P - P_b) \qquad (A\text{-}2)$$

Where m and n' are the slopes of the oil formation volume factor and solution gas-oil ration curves, respectively.

If a reservoir at standard conditions is considered, the oil density at reservoir conditions can be written as

$$\rho_{or} = \rho_{os}/B_{os} \qquad (A\text{-}3)$$

In general for any reservoir the oil density at the bubble point pressure can be calculated using the following equation

$$\rho_{ob} = \frac{\rho_{or}\, B_{os} + 0.1781\, R_{sb}\, \rho_g}{B_{ob}} \qquad (A\text{-}4)$$

Where $\rho_{or}\, B_{os}$ is the oil density st standard conditions and $0.1781\, R_{sb}\, \rho_g$ is the amount of gas per unit volume of oil, measured at standard conditions.

Using the oil density value at the bubble point pressure, the oil density at any other pressure can be computed by

$$\rho = \frac{\rho_{ob}\, B_{ob} - 0.1781\,(R_{sb} - R_s)\rho_g}{B_o} \qquad (A\text{-}5)$$

Where $0.1781\,(R_{sb} - R_s)\rho_g$ is the amount of gas per unit volume of oil which comes out of solution as a result of the pressure change from P_b to P, measured at standard conditions.

Combining Eqs. (A-1), (A-2) and (A-5) we get, finally:

$$\rho = \frac{\rho_{ob}B_{ob} - 0.1781\{R_{sb} - [R_{sb} + n'(P-P_b)]\}\rho_g}{B_{ob} + m\,(P - P_b)} \qquad (A\text{-}6)$$

for

$$P \geq P_1$$

thus

$$\rho = \frac{\rho_{ob}\, B_{ob} + n\,(P - P_b)\,\rho_g}{B_{ob} + m\,(P - P_b)} \qquad (A\text{-}7)$$

for $P \geq P_1$

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where $n = 0.1781\ n'$.

APENDIX B

MODIFIED FLOW EQUATION

Equation (3)

$$\rho = \frac{\rho_{ob} B_{ob} + n (P-P_b) \rho_g}{B_{ob} + m (P-P_b)} \qquad (3)$$

can be solved for $P-P_b$

$$P - P_b = \frac{(\rho - \rho_{ob}) B_{ob}}{(n\rho_g - m\rho)} \qquad (B\text{-}1)$$

Differentianting Eq. (C-1) with respect to some arbitrary coordinate s, say, yields

$$\frac{\partial \rho}{\partial s} = \frac{B_{ob} (n\rho_g - m\rho_{ob})}{(n\rho_g - m\rho)^2} \frac{\partial \rho}{\partial s} \qquad (B\text{-}2)$$

or simply

$$\nabla_p = \frac{B_{ob} (n\rho_g - m\rho_{ob})}{(n\rho_g - m\rho)^2} \nabla\rho \qquad (B\text{-}3)$$

Substituting Eq. (C-3) into Eq. (4) the flow equation in term of density is obtained:

$$\nabla \{ \rho \frac{k}{\mu} [\frac{B_{ob} (n\rho_g - m\rho_{ob})}{(n\rho_g - m\rho)^2} \nabla\rho + \rho g \nabla h] \} - G$$

$$= - \phi \frac{\partial \rho}{\partial t} \qquad (B\text{-}4)$$

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TABLE No. 1

	GROUP A		GROUP B		GROUP C	
	CASE 1	CASE 2	CASE 1	CASE 2	CASE 1	CASE 2
Bubble point (Pb), lppc	1500	1995	3060	4305	4064	1904
Near Boundary Pressure (P_1), lppc	504	1025	2006	2027	2508	1001
Gas in Solution at Pb (Rs), SCF/STB	402	893	1213	1279	2860	3195
Oil Volumetric Factor at Pb ($B_o b$), RB/STB	1.210	1.539	1.588	1.712	2.480	3.384
Slope of Bo vs. P (m) (x 10^4) RB/STB/lppc	0.802	1.497	1.671	1.419	4.859	10.881
Slope of Rs vs P (n'), SCF/STB/lppc	0.202	0.345	0.371	0.312	0.907	1.626
Oil Gravity at s.c. (API)	26.4	39.7	42.9	45.2	42.9	49.5
Specific Gravity of Gas (s.c.)	.90	.90	.81	.85	.80	1.1
Oil Compressibility Factors (Co) (x 10^5)						
a) Slightly Compressible Fluid	2.741	3.376	4.813	3.387	10.461	9.921
b) Ideal Fluid	2.704	3.274	4.695	3.262	9.9928	9.503

TABLE No. 2

MAXIMUM DEVIATION

	GROUP A				GROUP B				GROUP C			
	CASE 1		CASE 2		CASE 1		CASE 2		CASE 1		CASE 2	
	I	SC	I	SC	I	SC	I	SC	I	SC	I	SC
E_{AP} (lppc)	7	12	11	16	14	25	47	83	61	107	36	52
E_{AT} (lppc)	4.1	7.7	6.5	10.6	8.1	14.9	30.9	55.8	37.2	70.9	23.2	33.4
E_{AN} (Dim.)	.0040	.0077	.0065	.0108	.0077	.0142	.0136	.0245	.0239	.0456	.0246	.0370
E_{PP} (%)	.58	.97	.60	.99	.50	.90	.105	2.38	1.70	3.17	2.02	3.15
E_{PT} (%)	.33	.70	.38	.64	.29	.60	.73	1.50	1.03	2.03	1.36	2.15
E_{PN} (%)	.33	.70	.39	.66	.28	.53	.37	.71	.66	1.38	1.50	2.34

I - Ideal Fluid

SC - Slightly Compressible Fluid

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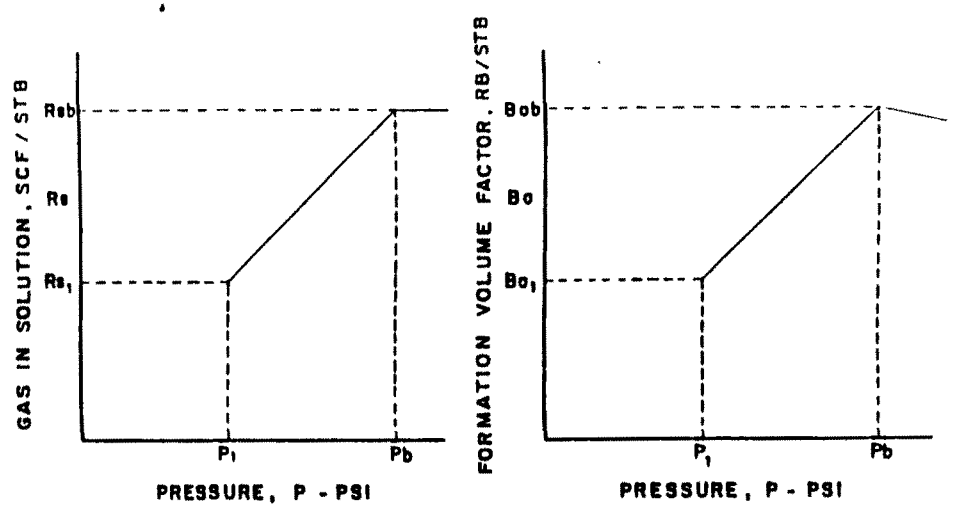

Figure 1: TYPICAL CRUDE PVT PROPERTIES

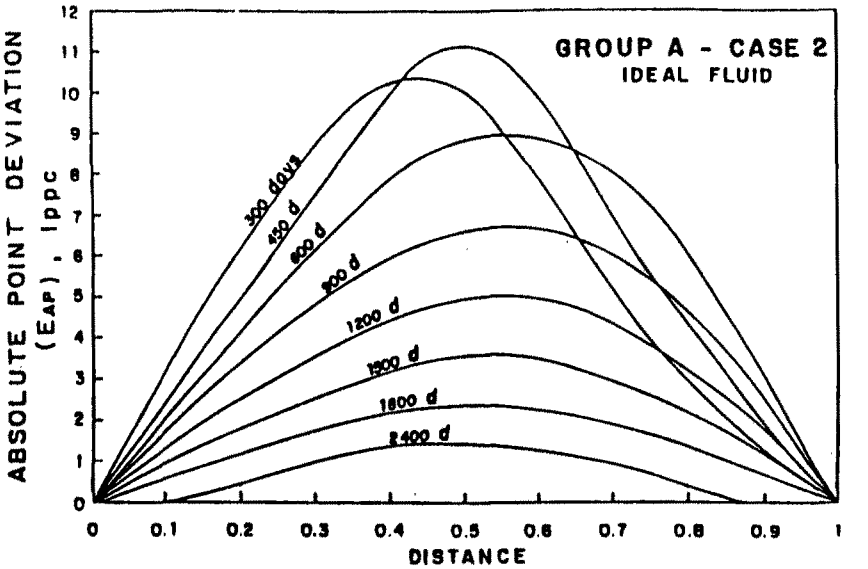

Figure 2: DEPENDENCE OF ABSOLUTE POINT DEVIATION ON TIME

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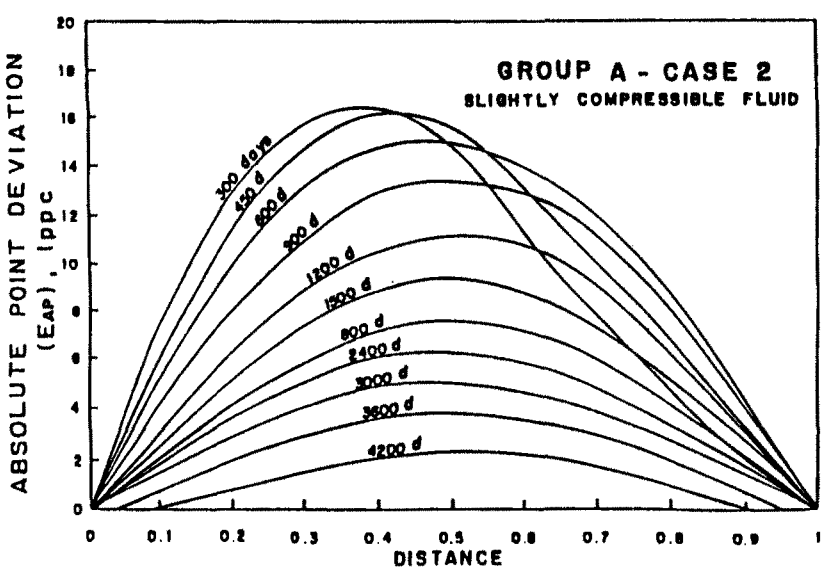

Figure 3: DEPENDENCE OF ABSOLUTE POINT DEVIATION ON TIME

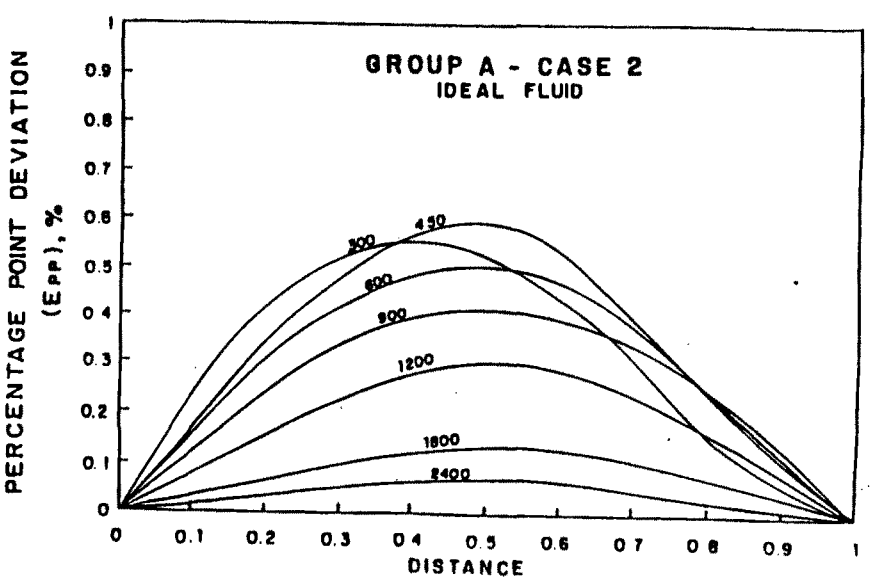

Figure 4: DEPENDENCE OF PERCENTAGE POINT DEVIATION ON TIME

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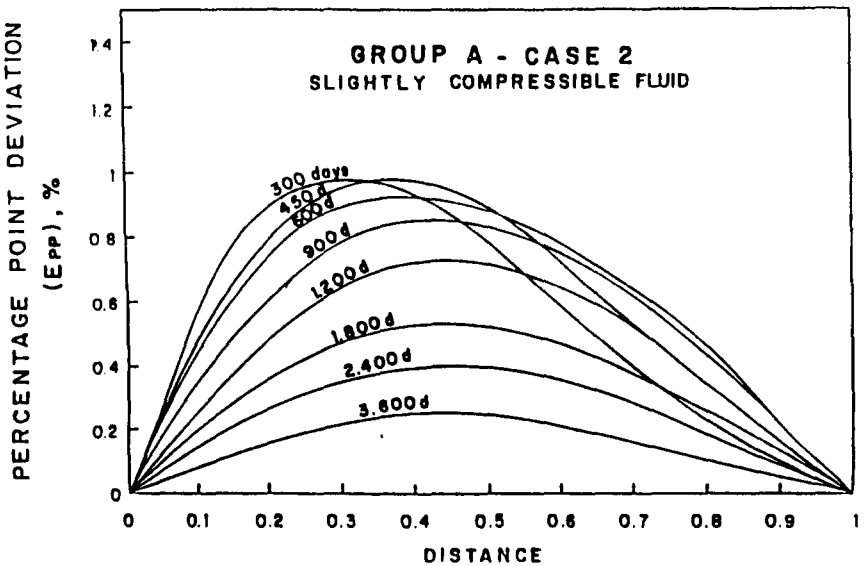

Figure 5: DEPENDENCE OF PERCENTAGE POINT DEVIATION ON TIME

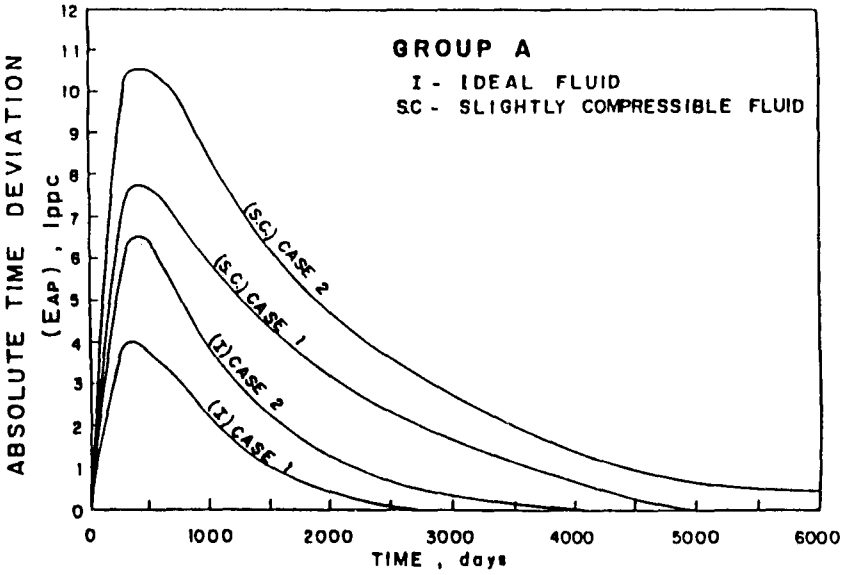

Figure 6: DEPENDENCE OF ABSOLUTE TIME DEVIATION ON TIME

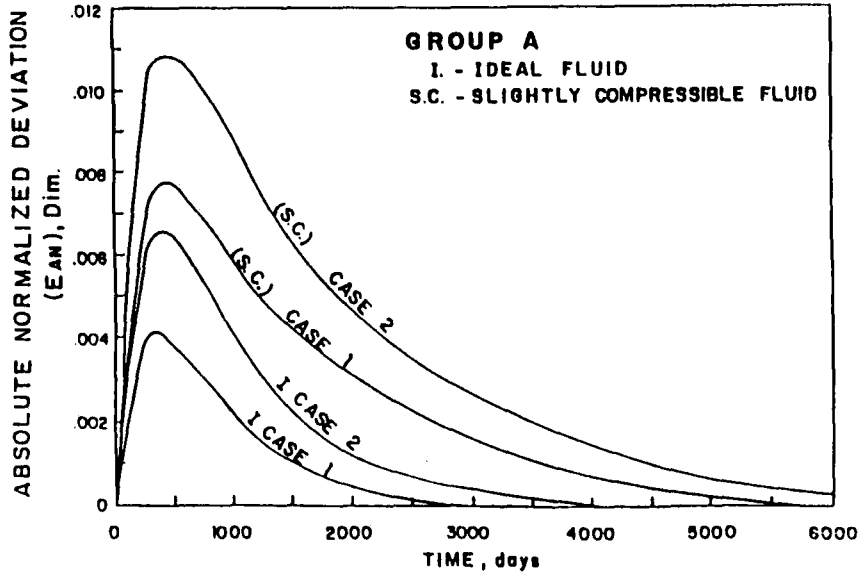

Figure 7: DEPENDENCE OF ABSOLUTE NORMALIZED DEVIATION ON TIME

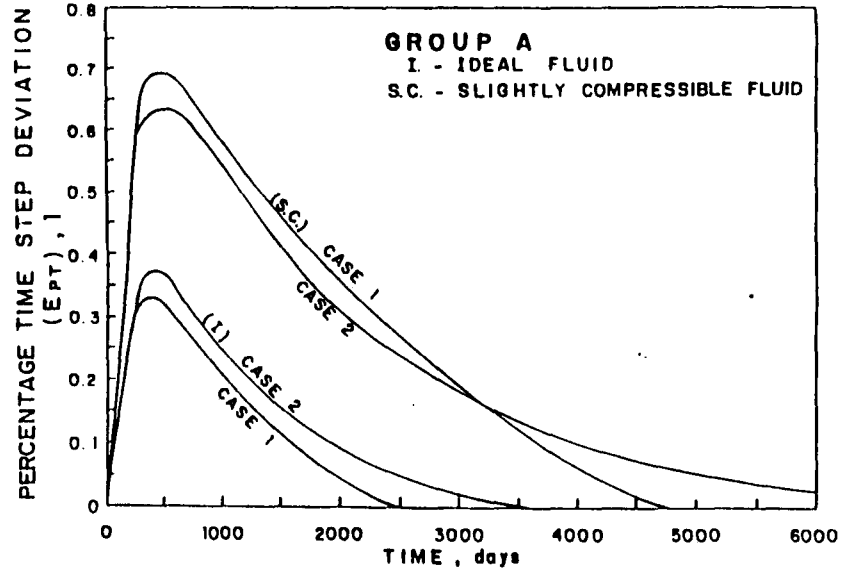

Figure 8: DEPENDENCE OF PERCENTAGE TIME STEP DEVIATION ON TIME

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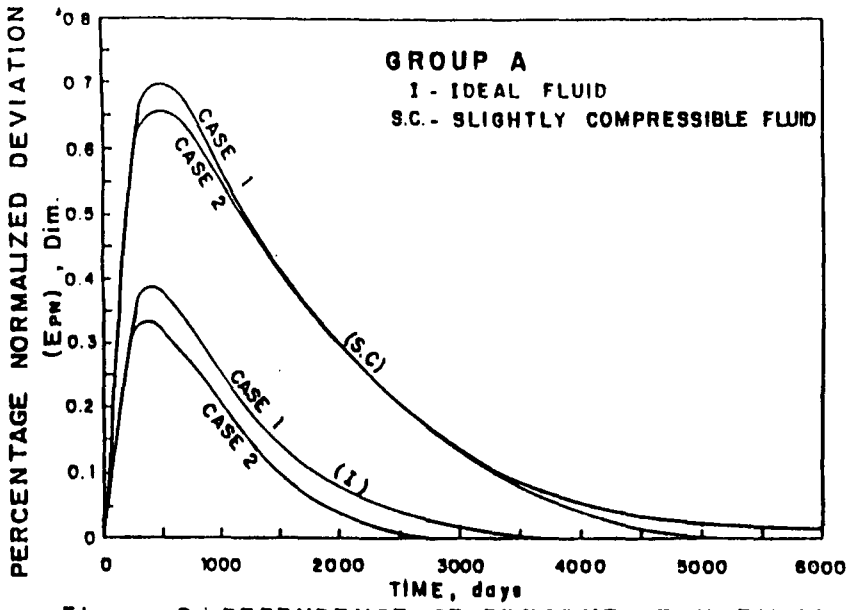

Figure 9 : DEPENDENCE OF PERCENTAGE NORMALIZED DEVIATION ON TIME

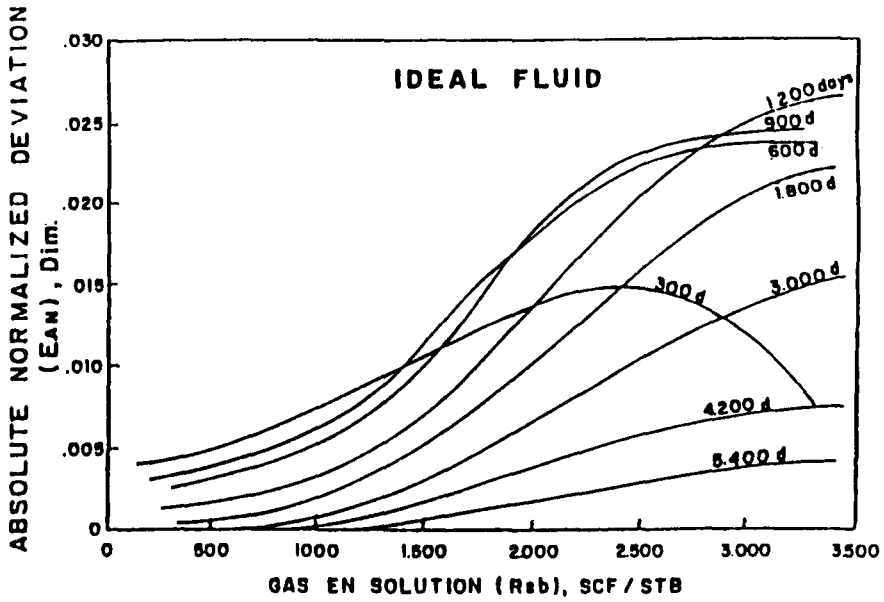

Figure 10 : DEPENDENCE OF ABSOLUTE NORMALIZED DEVIATION ON GAS IN SOLUTION

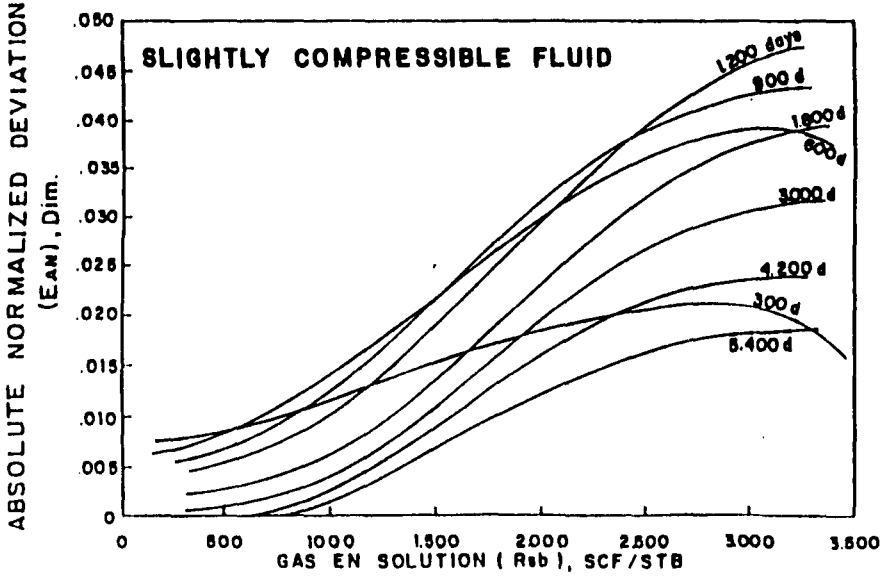

Figure 11 : DEPENDENCE OF ABSOLUTE NORMALIZED DEVIATION ON GAS SOLUTION

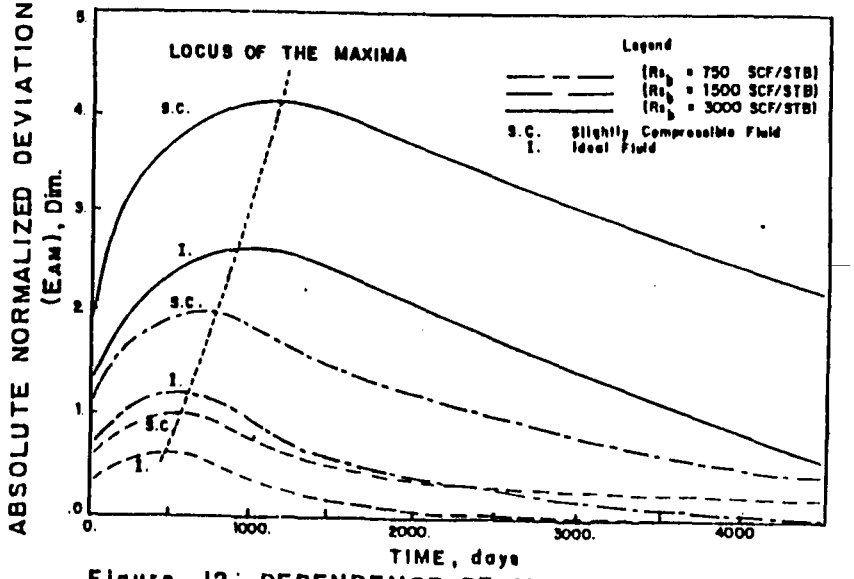

Figure 12 : DEPENDENCE OF ABSOLUTE NORMALIZED DEVIATION ON TIME

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